（2） Summary of Previous Research

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I provide backgrounds and summaries of the following two peer-reviewed papers.

peer-reviewed paper [1] : S. Ohno. *Rarita-Schwinger fields on nearly parallel G_2-manifolds.*

peer-reviewed paper [2] : S. Ohno, T. Tomihisa. *Rarita-Schwinger fields on nearly Kähler manifolds.*

paper in preparation [3] : Y. Homma, N. Imada, and S. Ohno. *Higher spin Killing spinors in 3-dimensional manifolds.*

Background of Previous Research

In classical spin geometry, much attention has been devoted to the Dirac operator on the spinor bundle $S_{1/2}$ and to the associated special spinor fields. In particular, harmonic spinors—elements of the kernel of the Dirac operator—and Killing spinors, which may be regarded as spinorial analogues of Killing vector fields, constitute central topics. On the other hand, there has been growing interest in the Rarita–Schwinger operator and Rarita–Schwinger fields on the spin-$k/2$ spinor bundle $S_{k/2}(k \in \mathbb{N})$, which can be regarded as higher-spin analogues of the Dirac operator and harmonic spinors. Whereas the existence or non-existence of harmonic spinors is controlled by the curvature, that of Rarita–Schwinger fields depends strongly on the underlying geometric structure and the dimension of the manifold. This motivates the question: on which classes of manifolds, and under what geometric conditions, can Rarita–Schwinger fields be explicitly characterized? Manifolds admitting Killing spinors form one particularly compelling class in this context.

Despite the importance of Killing spinors, their higher-spin counterparts have not been systematically developed. Formulating higher-spin Killing spinors and investigating their properties and existence is therefore a natural and significant direction in higher-spin spin geometry.

Background of Research results

(1) **Studied spin-3/2 Rarita-Schwinger fields on nearly parallel G_2-manifolds** (peer-reviewed paper [1])

Nearly parallel G_2-manifolds form a class of manifolds admitting Killing spinors, and are 7-dimensional Einstein manifolds with positive scalar curvature. By deriving several identities associated with a "good connection" on a nearly parallel G_2-manifold, I showed that on any compact nearly parallel G_2-manifold the space of Rarita–Schwinger fields is isomorphic to a subspace of a certain eigenspace of the Laplacian. As a consequence, one can conclude that Rarita–Schwinger fields do not exist on several nearly parallel G_2-manifolds. On the other hand, Homma and U. Semmelmann have shown that Rarita–Schwinger fields exist on many torsion-free G_2-manifolds. This indicates that the behavior of Rarita–Schwinger fields can be drastically different even among manifolds whose structure group reduces to the same group.

(2) **Studied spin-3/2 Rarita-Schwinger fields on nearly Kähler manifolds** (peer-reviewed paper [2])

Nearly Kähler manifolds form another class of manifolds admitting Killing spinors; in particular, they are Einstein manifolds with positive scalar curvature. A "good connection" with properties analogous to the one used in (1) for nearly parallel G_2-manifolds also exists on nearly Kähler manifolds. Accordingly, following the same strategy as in the nearly parallel G_2-case, I derived a collection of identities associated with this good connection and proved that, on any compact 6-dimensional nearly Kähler manifold, the space of Rarita–Schwinger fields is isomorphic to the space of harmonic 3-forms. The manifold $S^3 \times S^3$ admits nearly Kähler structures, both homogeneous and non-homogeneous, and in these cases there exists a 2-dimensional space of Rarita–Schwinger fields. In contrast, for the standard product metric, one can show that no Rarita–

Schwinger fields exist. This provides, to the best of my knowledge, a first example demonstrating that the existence of Rarita–Schwinger fields depends not only on topology but also on the choice of metric, and is therefore of particular significance.

(3) **Studied higher spin Killing spinors in 3-dimensional manifolds** (paper in preparation [3])

Killing spinors enjoy several fundamental properties: for instance, any manifold admitting a Killing spinor is necessarily Einstein, and Killing spinors appear precisely as the spinors attaining equality in eigenvalue estimates for the Dirac operator. Motivated by these facts, I introduced higher spin Killing spinors, a higher-spin analogue of Killing spinors, on spin manifolds and investigated their properties and existence in dimension three. More precisely, I proved that (i) any 3-dimensional manifold admitting a higher Killing spinor must be Einstein—and hence of constant sectional curvature; (ii) any spinor attaining equality in an eigenvalue estimate for the Rarita–Schwinger operator is a higher Killing spinor; and (iii) higher Killing spinors can be constructed explicitly on the 3-sphere.