Results of my research

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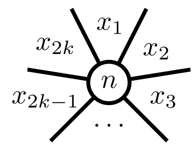
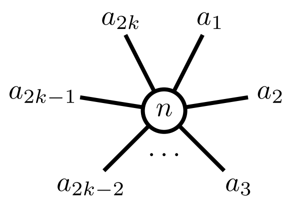
Let G be a planar Eulerian graph whose vertices are assigned integer values. For each vertex of G, consider the system of linear equations over the integers that assigns integer values to the planar regions incident to that vertex and satisfies the relation shown below. We call this system the integral region choice problem and denote it by $P(G)$.



$$x_1 + x_2 + \cdots + x_{2k} = -n$$

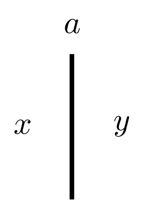
Kawamura introduced Z-colorings, that is, assignments of integers to the edges of planar 4-valent graphs (link projection diagrams), and gave a diagrammatic method for solving the corresponding $P(G)$. Okoshi and I generalized this result to $P(G)$ for planar Eulerian graphs.

The generalization of Kawamura's Z-coloring to planar Eulerian graphs is given as follows. The integer values on the edges around each vertex satisfy the relation shown below.



$$a_1 + a_3 + \cdots + a_{2k-1} = -n$$
$$a_2 + a_4 + \cdots + a_{2k} = -n$$

Kawamura proved that, in the case of planar 4-valent graphs, G is Z-colorable if and only if $P(G)$ is solvable. Moreover, when $P(G)$ is solvable, he gave an algorithm to obtain a Z-coloring of G, and showed that a solution to $P(G)$ can be obtained from a Z-coloring by assigning integer values to regions so as to satisfy the relation shown below. We generalized these results to planar Eulerian graphs.



$$x + y = a$$