

【A summary of past research achievements (Saki Okuhara)】

My research interest is the study of special Lagrangian submanifolds from the viewpoint of harmonic maps and the theory of integrable systems. Special Lagrangian submanifolds are real submanifolds of Calabi-Yau manifolds, defined by certain class of differential forms called the calibrations. Also, they are minimal Lagrangian submanifolds of Calabi-Yau manifolds. It implies that we can observe them from various points of view.

The connection between special Lagrangian submanifolds and integrable systems can be seen in the case of the 3-dimensional complex Euclidean space \mathbb{C}^3 as the ambient space (D. Joyce, *Special Lagrangian 3-folds and integrable systems*, 2008).

Special Lagrangian cones induce minimal Lagrangian immersions from Riemann surfaces to the 2-dimensional complex projective space $\mathbb{C}P^2$. Conversely, we can obtain (immersed) special Lagrangian cones from minimal Lagrangian immersions $\psi : S \rightarrow \mathbb{C}P^2$ of Riemann surfaces S .

My first result is the construction of minimal Lagrangian immersions $\psi : S \rightarrow \mathbb{C}P^2$ via the *generalized Weierstrass representation*, which is the theory about the construction of harmonic maps (J. Dorfmeister, F. Pedit and H. Wu, *Weierstrass type representation of harmonic maps into symmetric spaces*, 1998). More precisely, we can obtain harmonic maps of S into symmetric spaces for a certain class of matrix-valued 1-forms on S as their initial conditions through that theory. I gave in my research a description of initial conditions for harmonic maps $\psi : S \rightarrow \mathbb{C}P^2$ to be minimal Lagrangian immersions. It immediately implies that we have special Lagrangian cone of \mathbb{C}^3 at the same time.

Harmonic maps $\psi : S \rightarrow \mathbb{C}P^2$ correspond to solutions of the affine Toda equations, one of the famous integrable systems (J. Bolton, F. Pedit and L. M. Woodward, *Minimal surfaces and the affine Toda field model*, 1995). In case that $\psi : S \rightarrow \mathbb{C}P^2$ are minimal Lagrangian, the corresponding solutions reduce to solutions of the Tzitzéica equation. Moreover, they reduce to that of the third Painlevé equation when they are radially-invariant. Focusing on this fact, I gave an example of matrix-valued 1-forms on \mathbb{C} as initial conditions which induce radially-invariant solutions of the third Painlevé equation, hence solutions $w : (0, +\infty) \rightarrow (0, +\infty)$ of the third Painlevé equation.

Now I am studying with T. Sakai (Tokyo metropolitan university) an analogy of the previous result for the complex quadratic cone Q_n^0 in \mathbb{C}^{n+1} . K. Hashimoto (Osaka city university) and T. Sakai showed that the complex cone Q_0^n admits a Ricci-flat Kähler structure (K. Hashimoto and T. Sakai, *Cohomogeneity one special Lagrangian submanifolds in the cotangent bundle of the sphere*, 2012). It would be expected that we can have a parallel discussion in case of Q_0^n .