

Future research

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(1) On the relation between 2-Calabi-Yau triangulated categories and generalized q -deformed rational numbers.

This study considers the $(n + 1)$ -braid group action on the 2-Calabi-Yau triangulated category \mathcal{C}_n and aims to clarify the combinatorial structure of spherical objects on \mathcal{C}_n . Specifically, we construct generalized q -deformed rational numbers (q -deformation of rational points in an $(n - 1)$ -dimensional projective space). I will clarify the relation between the $(n + 1)$ -braid group action on \mathcal{C}_n and the $PSL_q(n, \mathbb{Z})$ -action on the set of generalized q -deformed rational numbers and extend our previous results on q -deformed rational numbers to generalized q -deformed rational numbers.

(2) Special values of the q -deformed modular group and arithmetic decomposition problem for q -deformed rational numbers

For an n -th primitive root of unity ζ_n , we define a kind of group which gives by taking $q = \zeta_n$ for $PSL_q(2, \mathbb{Z})$. We will study its construction. On the other hand, it is conjectured that if the denominator of an irreducible fraction is prime, the denominator polynomial of its q -deformed rational number is irreducible in $\mathbb{Z}[q]$, and moreover, the same holds for the left q -deformed rational numbers. This has been checked by computer for primes below 730, but in general it remains open. As an example for the case left q -deformed rational numbers, for a prime p , $q^p + q^{p-2} + \dots + q + 1$ is expected to be irreducible in $\mathbb{Z}[q]$, we resolved several cases. We will consider the remainder cases and the case of q -deformed rational numbers.