Summary of Research Accomplishments

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My research is in complex geometry, with "K-stability" as a key concept. K-stability is an algebro-geometric notion introduced by Tian and Donaldson as a condition for polarized manifold to admit canonical Kähler metrics. The Yau–Tian–Donaldson conjecture, which states that "the existence of constant scalar curvature Kähler metrics on polarized manifolds is equivalent to K-stability," has been actively studied.

Summary of "K-instability of hyperplane sections of Segre Varieties"

Let X be a hypersurface of bidegree $(1,1)$ in $\mathbf{P}^m \times \mathbf{P}^n$, which is also a hyperplane section of the Segre variety. In the 1980s, Sakane and Hano proved that when $m \neq n$ and X is smooth, there exist no constant scalar curvature Kähler metrics in any Kähler class on X. Inspired by this result, I computed the Futaki invariant of X using a purely algebraic way and prove that when $m \neq n$, (not necessarily smooth) X is K-unstable with respect to any polarization.

Summary of "Examples of relatively Ding unstable Calabi dream manifolds"

In this paper, we prove that uniform relative Ding stability is not preserved under direct products in general and construct examples of manifolds that are uniformly relatively K-stable for any polarization but relatively Ding unstable in all dimensions greater than two.

Summary of "A note on the Yotsutani-Zhou condition for relative K-instability"

Yotsutani-Zhou found a criterion for toric Fano manifolds to be relatively K-unstable with respect to anticanonical polarization and completely determined relatively K-stable three-dimensional toric Fano manifolds. In this paper, we reveal a simple necessary condition for applying the Yotsutani-Zhou criterion and show that there are actually no three-dimensional examples where their criterion applies.

Summary of "Relative Ding and K-stability of toric Fano manifolds in low dimensions"

In this paper, we calculate Mabuchi constants for all toric Fano manifolds in dimensions up to four and completely determine which are uniformly relatively Ding stable.

Summary of "Stability of anti-canonically balanced metrics"

"Anticanonically balanced metrics" were introduced by Donaldson as geometric quantization of Kähler-Einstein metrics. In this paper, we introduce "F-stability" by considering a functional with anti-canonically balanced metrics as critical points, and prove the following statements: "existence of anticanonically balanced metrics implies F-stability", "K-stability with respect to anticanonical polarization implies asymptotic F-stability" and "asymptotic Chow stability implies asymptotic F-stability. "