

## Future Research Plan

### (I) Vacuum Structure of Extra Dimensions and the Generation Structure of Quarks and Leptons

The discovery of the Higgs boson in 2012 firmly established the Standard Model of particle physics. Nevertheless, the Standard Model leaves many mysteries unresolved and is not regarded as a final theory. For example, “Why do quarks and leptons appear in three identical copies?” is known as the generation-number problem. Likewise, “Why do the masses of quarks and leptons increase exponentially with each generation?” is known as the mass-hierarchy problem. These questions cannot be answered within the framework of the Standard Model.

This research aims to address the generation-number and mass-hierarchy problems by considering higher-dimensional theories with extra dimensions. The distinctive feature of this project —unlike other approaches— is that it does not attempt to solve the problem by adjusting theoretical parameters by hand. Instead, it seeks to determine the vacuum structure dynamically by minimizing the quantum-corrected vacuum energy, thereby providing a natural resolution of both the generation-number and mass-hierarchy problems.

Specifically, as described in the “Summary of Previous Research Achievements,” we will investigate higher-dimensional models featuring point-like interactions in the extra dimensions. By computing the vacuum (Casimir) energy in such models and determining the configuration that minimizes this energy (i.e., the vacuum configuration), we intend to dynamically fix the parameters that determine the generation structure of quarks and leptons.

### (II) A Discrete Stochastic-Process Approach to Quantization Without the Need for a Continuum Limit

Stochastic quantization is a third approach to quantization, distinct from canonical quantization and the path-integral method. In particular, the Parisi-Wu method realizes the quantization of field theories by introducing a continuous fictitious time and formulating a Langevin equation. This framework has significant potential, with extensibility and applicability even to gauge theories. For numerical applications, however, the fictitious time in the Langevin equation must be discretized with a time step  $\varepsilon$ . Conventionally, one must ultimately take the continuum limit  $\varepsilon \rightarrow 0$ , which poses an obstacle to computational efficiency in lattice simulations. If a formulation of lattice computations could be developed that does not require the  $\varepsilon \rightarrow 0$  limit, it would open the door to major new developments with wide-ranging applications. The aim of this research is to establish an entirely new quantization method based on discrete stochastic processes that eliminates the traditionally indispensable continuum limit, and to apply it to quantum field theory and lattice numerical analysis in order to elucidate the dynamics of fields and spacetime. To this end, the project will address the following concrete objectives: Formulating lattice gauge theories and establishing efficient numerical computational techniques; Constructing quantum algorithms based on discrete stochastic processes; Clarifying the connections with extra-dimensional theories and supersymmetry.

### (III) Torus Models with Uniform Magnetic Flux

Systems consisting of an  $n$ -dimensional torus with a uniform magnetic flux are of great interest not only for models of extra dimensions in particle physics, but also for condensed-matter systems and from a mathematical standpoint. Known characteristic properties of such systems include: (1) A classical translational symmetry that is broken quantum mechanically to a discrete translational symmetry; (2) Quantization of magnetic flux; (3) The appearance of massless chiral fermions; (4) Nonzero vacuum expectation values of scalar fields necessarily acquire coordinate dependence. Among these, properties (1) and (4) were uncovered through our work, revealing that the system possesses a topologically nontrivial vacuum structure. Indeed, we have shown that the vacuum of the system exhibits a vortex configuration. The objective of the present research is to provide a theoretical elucidation of the detailed structure of this vacuum.