Research Plan

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1 Normal Form Theory of Difference Equations

In collaboration with S. Hirose, Yoshitsugu Takei and Yumiko Takei, toward general theory of the exact WKB analysis of difference equations (,or systems of difference and differential equations), we have been analyzing some concrete example. I, in particular, am interested in analytic foundation of normal form theory at a logarithmic turning point, which is unique to difference equations. A logarithmic turning point was found by Yoshitsugu Takei et al., and it is coming to light by our resent study that it appears universally in difference equations. Analysis of a logarithmic turning point and normal form theory at it are expected to be a central part of the exact WKB analysis of difference equations. I proceed to establish analytic foundation of the normal form theory, while continuing analysis of concrete examples.

Also, if we consider a system of difference and differential equations, a logarithmic turning point is naturally associated to a double turning point of a differential equation. Working toward normal form theory of such systems, I reconsider some problems in the normal form theory at a double turning point of a differential equation, such as normalization of solutions and the connection formula.

2 Analytic Foundation of the Exact WKB Analysis of Painlevé Equations

The exact WKB analysis of Painlevé equations was vigorously studied by Aoki, Kawai, Takei, et. al., and many interesting results are known. However there are formal solutions and formal transformations in the theory, for which we need to give analytic foundation.

As for formal transformations, Borel summability of the formal transformation of associated linear equations is partly established, I try to solve this problem completely. As for formal solutions, summability of 0-parameter solutions (formal power series solutions) and 1-parameter solutions (transseries solutions) is known, but analytic interpretation of 2-parameter solutions is still open. From classically known results, 2-parameter solutions are expected to be convergent and to have asymptotically elliptic behavior. Some result about this expectation is being obtained for the simplest equation, i.e., the first Painlevé equation. I try to reach the elliptic behavior for the first Painlevé equation, while proceeding to calculate for other Painlevé equation.

3 Exact WKB Analysis of Higher-Order Painlevé Equations

It is known that the exact WKB analysis of higher-order linear equations, in particular Borel summability of WKB solutions, is extremely difficult. Likewise, summability of formal solutions of higher-order Painlevé equations is believed to be very difficult. Meanwhile, many higher-order Painlevé equations are known to be obtained by restriction of multivariable equations. In such a case, the problem of Borel summability of formal solutions may get easier, and I consider Borel summability for such equations.

Some higher-order Painlevé equations have, as associated linear equations, second-order equation, and in that case, transformation theory near a turning point may be handled in a similar manner to the transformation theory of second-order linear equations. As for transformation theory, we also aim to deal with such higher-order Painlevé equations.