Research Achievements

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My research theme is the exact WKB analysis, a kind of asymptotic analysis of singularly perturbed differential eqautions. Especially, I have been studying fundamental problems such as Borel summability (a way of giving meaning to divergent series) of WKB solutions (formal power series solutions w.r.t. the singular-perturbative parameter) and Stokes phenomena. I have solved some related problems such as classification of Stokes graphs, Borel summability of transformation to canonical equations. Also, in addition to such fundamental problems, I have been interested in application of the exact WKB analysis to physics, and have done some works. (Recently, I have been working in the exact WKB analysis of Painlevé equations and difference equations. For such (ongoing) works, see Research Plan.)

In [9], I studied classification of Stokes graphs of second-order equations on the Riemann sphere. It had been conjectured that some properties would characterize the Stokes graphs, and I proved it in general. Also, I developed an algorithm to generate Stokes graphs in order from those with less vertices and enumerated Stokes graphs. From the resulting sequence, I observed a relation with other fields. In the process of this study, I found a gap in the proof of properties of Stokes graphs, and remedied it.

In transformation theory to canonical equations, I proved Borel summability of the formal power series which in a neighborhood of a double turning point transforms the equation to the canonical equation (degenerate Weber equation) ([3]). By this result, e.g., a proof of the connection formula related to double turning points is given([2]). Also, by applying the method of the proof, Borel summability of other kinds of transformation series (e.g., one involving two or more simple turning points) is proven similarly; While transformation theory in a neighborhood of a single turning point is related to Stokes phenomena as the independent variable moves, transformation theory in a neighborhood of two or more turning points is related to Stokes phenomena as a parameter varies. Both are important, and Borel summability in the latter problem is now shown in a unified manner([1], cf. also [8]).

Borel summability of WKB solutions and Stokes phenomena are inseparable, and as a research toward an extremely difficult problem, Borel summability of WKB solutions of higher-order equations, I analyzed a Stokes phenomenon (with respect not to the singular-perturbative parameter but to another parameter) of a particular higher-order equation. I described the Stokes phenomenon (connection formula) explicitly, and at the same time developed tools for analysis of more general Stokes phenomena([5]).

In higher-order equations which are obtained as sections of some holonomic systems of several variable, there appear some kind of double turning points called non-hereditary turning points. Using the steepest descent method or its generalization, the exact steepest descent method, I analyzed Stokes phenomena caused by such turning points([4,10]).

As a physical problem related to the problems above, I studied, by using the exact WKB analysis, non-adiabatic transition of multilevel systems([6,7]). In a preceding research, it was shown that so-called new Stokes curves (NSCs for short), which are peculiar to higher-order equations, do not have any effect on the calculation of transition probabilities. In another preceding research, a degenerate situation, which was excluded in the research above, was studied with a complex perturbation, and it suggested that NSCs could affect on transition probabilities. I further studied other examples, and showed that we cannot ignore the effects of NSCs (numerically or approximately). On the way of the study, I encountered a bifurcation phenomenon of Stokes curves at a double turning point. I analyzed this phenomenon, and described the change of Stokes multiplier before and after the bifurcation.