Future research plan

Takuya Sato

- **Optimal L^2(mass)-decay for dissipative nonlinear Schrödinger equations:**

On the dissipative nonlinear Schrödinger equations, the relation between spatial regularity and time mass decay of solutions was revealed by previous works, however it does not be revealed a sufficient condition for solutions with the optimal mass decay. Kim-Sunagawa (2014) showed the optimal mass decay of solutions for nonlinear dissipative Klein-Gordon equation by considering the nature of the finite propagation speed of associated wave functions. On the other hand, the infinite propagation speed of solutions for nonlinear Schrödinger equations cause a difficulty to analysis obtaining the optimality of the mass decay. Therefore I aim to show the essential difference between the long time behavior of solutions to the Klein-Gordon and Schrödinger equation. To this end, we carefully check regularity and the shape of solutions by analyzing method obtained by Li-Nishii-Sagawa-Sunagawa (2022) and using the argument using the Pseudo conformal transformation by Cazenave-Han-Naumkin (2021). If our aim is achieved, then it may be clarified the characteristic mechanism of dissipative and dispersive nature in the nonlinear Schrödinger equation.

- **Analysis of a dissipative nonlinear Schrödinger system:**

For the nonlinear Schrödinger system, the gauge invariance of nonlinearities stabilize the behavior of wave functions under a special relation on coefficients of dispersive term. This phenomenon is called "*resonance*" and it is important to consider the asymptotic behavior of solutions to the nonlinear Schrödinger equations. Our aim is to analysis the long time behavior of the nonlinear Schrödinger system without the resonance condition. To this end, we need to have the sharp analysis due to Li-Sunagawa (2016) and consider the reduced nonlinear ODE system associated with the system. In addition, we also utilize the power series expansion for the solution exhibiting real analyticity or Gevrey regularity in space valuable.

- **Clarifying the physical impacts of dissipative nonlinear Schrödinger equations**

For the mass-conserving nonlinear Schrödinger equation (NLS), the Madelung transform derives the Korteweg-Euler equations. Since they lack viscosity, these equations describe frictionless quantum fluids like Bose-Einstein condensates near absolute zero. Recently, "two-fluid models" mixing these quantum fluids with friction-bearing normal fluids have gained attention. My research applies the Madelung transform to dissipative NLS equations to derive the dispersive compressible Navier-Stokes equations, which incorporate viscosity. By linking dissipation to viscosity, I aim to mathematically clarify the dynamics of these two-fluid models.