

Future Research Plan

(1) Topological and computational approaches to eigenvalue problems

I aim to establish topological methodologies for eigenvalue problems that arise from stability analyses of solutions to reaction-diffusion systems and related equations in spatially higher-dimensional settings. I have been extending a topological framework known as stability index theory for such problems, and in the next stage I will attempt to extend this framework to traveling waves on Riemannian manifolds. In connection with this extension, I will investigate in detail how geometric properties of the underlying manifold—such as curvature—affect the stability of traveling waves. This direction can be viewed as an extension, from spot dynamics to traveling waves, of my previous work on the motion of spot solutions on curved surfaces.

Moreover, to compute an infinite-dimensional stability index concretely, it is necessary to discretize Laplace-type or more general elliptic differential operators in the transverse directions and to construct finite-dimensional approximations. Such approximations (e.g., spatial discretization or Galerkin methods) may, however, cause discrepancies in quantities such as the Fredholm index or the stability index, depending on the choice of discretization. To capture and understand these phenomena, I will study in detail the properties of elliptic-type operators defined on discretized networks/graphs, and analyze the behavior of the associated indices in the continuous limit.

(2) Stability and bifurcation theory for nonlocal reaction-diffusion equations

For reaction-diffusion equations with nonlocal terms described by convolution-type operators, I have shown that they can be approximated by reaction-diffusion systems that include advection and spatial-shifted terms. In particular, I clarified that the odd component of the convolution kernel generates phase velocity and group velocity, and as a consequence it has been revealed that spatially periodic traveling waves can emerge through a Turing bifurcation.