

Future Research (Hiroshige Shiga)

Regarding future research, I plan to continue along the lines of my previous work. Specifically, this involves the study of deformation spaces of Riemann surfaces and the study of Riemann surfaces themselves. In particular, I will focus on the ongoing research on the moduli space of generalized Cantor sets. A generalized Cantor set is determined by the countable infinite product Ω of open intervals $(0, 1)$. The current issue involves determining the set of all elements of Ω that yield Cantor sets quasi-conformally equivalent to the Cantor set determined by a given element ω of Ω . We refer to this set as the moduli space of ω . The goal is to determine this moduli space and clarify its structure.

This problem extends the research carried out in [1], [4], and [7]. What is particularly interesting is that Ω serves as an object of classical ergodic theory. Therefore, it is possible to attack this problem using tools from ergodic theory. While this approach has seen partial success so far, satisfactory results have not yet been achieved. Currently, we are exploring solutions by combining these methods with techniques from the theory of quasi-conformal mappings. We will also continue our study of quasicircles on Riemann surfaces. Quasicircles are generally defined in the complex plane, and various characterizations of them are known. Schnipers–Staubach extended this concept to Riemann surfaces and showed that, on closed Riemann surfaces, quasicircles can be characterized by a certain extension principle for harmonic functions with finite Dirichlet integral. We have further extended this to general Riemann surfaces and clarified its connection with the theory of quasiconformal deformations [3]. The aforementioned “extension principle for harmonic functions with finite Dirichlet integral” is, of course, known as a classical characterization of quasicircles. In the future, we intend to advance our research on the relationship between other characterizations of quasicircles and quasicircles on Riemann surfaces.

Another recent line of research focuses on the curve complex. This is a joint study with Mr. Ryo Matsuda, a graduate student at Kyoto University, and Ms. Kanako Oie, a graduate student at Nara Women's University. The study examines the distribution of the lengths of (tight) geodesics in the curve complex and is expected to be released as a preprint soon. This research represents not only a new area that I have not previously explored but also a fresh attempt at collaborative work with young researchers.

I have participated in and given presentations at research meetings on topology, geometry, and probability theory. Through these activities, I have aimed to broaden my knowledge beyond complex analysis into related fields. I hope to further expand these opportunities to deepen and diversify my research.