Shinichi Tajima

Research acomplishments

I have studied algebraic analysis and studied systems of partial differential equations on a complex manifold and complex analysis of singularities.

1.**Algebraic analysis of tangential Cauchy-Riemann equations and the problems of analytic continuation of holomorphic solutions to a system of partial differential equations**

Based on the theory of D-modules and the microlocal analysis, I considered the problem of analytic continuation with respect to a generic CR submanifolds for the complex of holomorphic solutions to a linear partial differential equations defined on a complex manifold. I showed, by using the langage of the derived category, that the obstructions of the analytic continuation can be described in terms of microlocal solution complex to the tangential system w.r.t. the CR submanifold. I showed in particular that the Zerner's result on analytic continuation for non-characteristic boundary is equivalent, from the microlocal point of view, to the Sato's regularity theorem of hyperfunction solutions to an elliptic system. Furthermore, by using the notion of a generalized Levi form, I generalized results due to Pallu de la Barrier and Tsuno. As applications of this approach, I studied H. Lewy phenomena, an extension of Bochner's tube theorem and some properties of CR hyperfunctions.

I demonstrated that the Henkin-Ramirez kernels give rise to a reproducing kernels to the CR microfunctions. I obtain a microlocalization of a Koppelman's homotopy formula for the Cauchy-Fantappier kernels, By regardeing the classical Bochner-Martinelli kernel as an object of derived category, I obtained a non vanishing theorem of the cohomology of the microfunction solution complex to the tangential Cauchy-Riemann systems. I constructed an integral representation formula and a fundamental solution to microfunction sotutions to tangential CR systems induced on certain kinds of pseudo convex boundaries.

2.**Adiabatic phase**

A localized state that arises in certain semi-conductor superlattice under an uniform electric field can be formulated as a singular pertabation problem for the Schrodinger equation with a finite gap potential. By using a multiple scale metod and elliptic functions, I explicitley computed an adiabatic phase factor of the reading term of an asysmptotic solution.

3. **Algebraic analysis of Grothendieck residues and algorithms**

By utilizing the theory of holonomic D-modules, I showed that the Grothendieck local residues can be computed. By introducing a concept of Noether operators for local cohomology classes, I constructed algorithms for computing Grothendieck local residues. The designed algorithms are implemented in a computer algebra system.

4. **Computational complex analysis of singularities**

I deviced an algorithm for computing local cohomology classes associated to a primary ideal in the rings of convergent power series. Based on the theory of Grothendieck local duality and the local cohomology, I constructed algorithms for computing ideal membership, normal forms, standard bases, ideal quotients in local rings and implemented these algorithms in a

computer algebra system. I introduced a new framework for treating ideals with parameters. As applications to singularity theory, I constructed and implemented algorithms for computing Tjurina number, μ^* invariants, limiting tangent spaces, logarithmic vector fields, Samuel multiplicity, generalized integral dependence relations and Bertini type invariants. I also introduced a framework for treating the Matlis duality. I have constructed algorithms for computing Neother operators to describe multiplicity structure of a primary ideals.

5. **Computational algebraic analysis of Bernstein-Sato polynomials**

By introducing a concept of comprehensive Groebner systems to PBW algebras, I derived an algorithm for computing s-parametric annihilators of polynomials with deformation parameters. I constructed an algorithm for computing b-functions associated to semi-quasi homogeneous polynomials. I have studied b-functions and the structure of relevant holonomic D-modules associated to hypersurfaces with non-isolated singularities introduced by de Jong and D. Siersma.