Research Plan

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KP and Toda hierarchies of B, C and D types Shortly after the KP hierarchy was introduced, its B- and C-type variants (the BKP and CKP hierarchies) were deviced from a Lie-theoretic perspective. Subsequent research led to the discovery of the D-type KP hierarchy (the DKP hierarchy) and a new B-type KP hierarchy (the large KP hierarchy). In recent years, A. Zabrodin et al. have proposed new types of Toda hierarchy (the B-Toda and C-Toda hierarchies). J.-P. Cheng et al. have also introduced various variants of the Toda hierarchy, and have pointed out their relationship to Zabrodin et al.'s BC-type Toda hierarchies. We would like to explore the detailed properties of these new integrable hierarchies, in particular, the description of solutions in terms of Grassmann manifolds.

Toroidal extension of soliton equations and higher dimensional integrable systems In recent years, the group led by Masashi Hamanaka has made significant progress in new research into higher dimensional integrable systems such as the anti-self-dual Yang-Mills equation. Toroidal extensions of soliton equations also share common characteristics with integrable systems derived from gauge theory. Methods for constructing solutions to these equations include Darboux transformations, the Cauchy matrix method, and direct integration. On the other hand, as in the case of soliton equations, an approach based on Grassmann manifolds is also possible. I hope to explore these relationships and gain new insights.

Quantum K-theory and relativistic Toda lattice In recent years, the group led by Takeshi Ikeda has been conducting research on the relationship between quantum cohomology and quantum K-theory of flag varieties and integrable systems. In particular, it is known that the relativistic Toda lattice emerges from quantum K-theory, and a deeper understanding of the relativistic Toda lattice is needed. Motivated by this, I will collaborate with Ikeda et al. to study the BCD-type relativistic Toda lattice.

Enumerative geometry and Frobenius manifolds B. Dubrovin formulated the geometric structure of genus-zero Gromov-Witten theory as a Frobenius manifold, and pointed out its relationship to integrable systems. A. Givental presented an expression for the generating functions of all-genus Gromov-Witten invariants in terms of operators on a bosonic Fock space. According to Givental, this operator representation may be thought of as quantization of a Lagrangian submanifold in infinite-dimensional symplectic geometry. I would like to understand its meaning and its impact on integrable systems.

History of mathematics Over the past few years, I have researched the 19th-century literature on topics such as the matrix-tree theorem, the origin of Grassmann manifolds, the interlace theorem for eigenvalues of real symmetric matrices and Sturm's theorem on the number of real roots of polynomials, and have found several interesting facts. Recently, I have become interested in the development of determinant theory in the 19th century, and would like to investigate papers of Cauchy, Jacobi, Cayley, Kronecker and Weierstrass who are known to have contributed to this field.