

On left-invariant metrics

For a Lie group G , denote by $\mathcal{M}_L(G)$ the set of all left-invariant metrics on G . I have shown that if an orbit of the action $\mathbb{R}_{>0} \times \text{Aut}(G) \curvearrowright \mathcal{M}_L(G)$ is an isolated orbit, then any left-invariant metrics contained in the isolated orbit are Ricci soliton.

The Ricci soliton is defined as a metric which gives a self-similar solution of the Ricci flow equation. Recent studies have revealed that if the orbit $\mathbb{R}_{>0} \times \text{Aut}(G) \cdot \langle, \rangle$ is an isolated orbit, then $\langle, \rangle$ gives a self-similar solution not only the Ricci flow equation but also various metric evolution equations. Hence I study these left-invariant metrics more deeply as follows:

- More examples of Lie groups G whose $\mathbb{R}_{>0} \times \text{Aut}(G)$ -actions have isolated orbits are required since they give nice examples for studying self-similar solutions of metric evolution equations. To find examples, 2-step nilpotent Lie groups are nice targets since their automorphism groups are relatively simple.
- A goal of my research is to classify Lie groups G which admit isolated $\mathbb{R}_{>0} \times \text{Aut}(G)$ -orbits. The generalized Alekseevskii conjecture has been proved by Böhm and Lafuente. As a consequence, a Lie group G which admits a left-invariant Ricci soliton $\langle, \rangle$ is isometric to a solvmanifold or a product space of a compact Einstein Riemannian Lie group and a flat abelian group. Recently, I have found some algebraic obstruction for an $\mathbb{R}_{>0} \times \text{Aut}(G)$ -orbit to become an isolated orbit. I try to classify solvable Lie groups G which admit isolated $\mathbb{R}_{>0} \times \text{Aut}(G)$ -orbits by applying the obstruction.

On arid submanifolds

I have generalized the essence of isolated orbit to arbitrary submanifolds as arid submanifolds. My future works on arid submanifolds will divide into two cases (i) homogeneous cases, (ii) inhomogeneous cases:

- (i) One can see that a homogeneous arid submanifold is an isolated orbit of some isometric action. One of the easy examples of isolated orbits is a non-principal orbit of a cohomogeneity one action. These isolated orbits are characterized as

a homogeneous submanifold whose full slice representation on the unit normal sphere is transitive. (★)

A goal is to classify submanifolds which satisfy (★) in Riemannian symmetric spaces. Cohomogeneity one actions on the space forms are well-understood. Hence, firstly, I observe (★)-submanifolds in the space forms, and try to classify in the general settings.

- (ii) No example of complete inhomogeneous arid submanifolds in homogeneous spaces seems to be known. One of the nice examples of inhomogeneous submanifolds are some isoparametric hypersurfaces in the sphere. Firstly, I want to investigate whether the focal submanifolds of isoparametric hypersurfaces in the sphere are arid submanifolds or not.