2 Research Plan

A central problem in complex geometry concerns the rigidity of embeddings of compact complex submanifolds. Given a compact complex variety C embedded in two complex varieties M and M' of the same dimension, one asks whether neighborhoods of C in M and M' are biholomorphically equivalent. While this problem is well understood in the presence of curvature, the flat case remains largely open, as classical geometric methods are ineffective.

In the flat setting, techniques from dynamical systems become essential. For embeddings of elliptic curves into complex surfaces, obstructions analogous to small divisor phenomena arise through cohomological equations governing linearization. Although these methods yield powerful analytic tools, the corresponding geometric conditions are difficult to characterize, and the structure of neighborhoods remains poorly understood when linearization fails. Related issues appear in the study of holomorphic foliations near compact complex submanifolds. While the case of curves in surfaces is well understood, higher-dimensional and higher-codimensional situations remain largely open beyond regimes accessible by linearization.

A further motivation comes from the study of singular positive metrics on nef line bundles. A nef line bundle may admit multiple singular positive metrics with distinct singularities, and understanding the minimality and regularity of such singularities is a fundamental problem. In particular, for compact Kähler manifolds with nef anticanonical bundle, it is natural to ask whether positivity implies regularity. Many results of Professor T. Koike show that singular positive metrics need not be smooth, and their behavior is closely related to dynamical conditions. It is conjectured that, except for specific cases, singular positive metrics on nef anticanonical bundles may exhibit arbitrarily severe singularities, though explicit examples remain scarce.

This project studies these questions through Ueda theory. One direction focuses on Ueda's problem for hyperelliptic manifolds and for general Hopf manifolds under weak geometric assumptions. These manifolds arise as finite quotients of complex tori or of primary Hopf manifolds, which were studied in a previous collaboration with Professor Laurent Stolovitch. The final direction applies these techniques to the study of singular metrics on anticanonical line bundles, with the aim of constructing examples exhibiting highly singular positive metrics and clarifying the role of Diophantine phenomena in this context.