

1 Past research achievements

My research lies in complex differential geometry and complex algebraic geometry, with a particular emphasis on positivity properties of holomorphic vector bundles and coherent sheaves, formulated in terms of cohomology theory, and on their geometric consequences.

A recurring theme in my work is to understand how weak notions of positivity—formulated analytically via singular Hermitian metrics and curvature currents—nevertheless impose strong global topological constraints on the underlying manifold or on holomorphic vector bundles. Methodologically, my research relies on tools from pluripotential theory, Monge–Ampère equations, and Bott–Chern cohomology, and aims to extend results that are well understood in the projective setting to the more general Kähler context. Moreover, some of the topological results I obtain remain valid even in the non-Kähler case, such as constructions of Chern classes for coherent sheaves, the characterization of the Kodaira–Iitaka dimension as a genuine limit rather than an upper limit, and holomorphic Morse inequalities involving tensor products with coherent sheaves. A central object of my recent research is the notion of a strongly pseudo-effective (strongly psef) vector bundle (introduced in the famous work of Boucksom–Demailly–Paun–Peternell) with a systematic study. This is an analytic positivity condition defined by the existence of singular Hermitian metrics whose curvature currents satisfy suitable semi-positivity properties in an appropriate weak sense. Compared with classical pseudo-effectivity, this notion is sufficiently strong to yield nontrivial geometric consequences, while remaining flexible enough to be compatible with analytic techniques such as positivity of direct image methods. In particular, in 2025, in joint work with collaborators, we obtain a complete classification of compact Kähler manifolds with nef anticanonical line bundle. This result generalizes earlier work of Demailly–Peternell–Schneider in the 1990s and of Cao–Höring in the 2010s. From a technical point of view, a key step in these classification results is the local triviality of certain fibrations, which ultimately reduces to the numerical triviality of an associated vector bundle. My main contribution is to show that a strongly psef vector bundle with trivial first Chern class is necessarily numerically flat in this context. To achieve this, I introduce the technique of Segre currents, which provides a bridge between the nefness of the anticanonical line bundle and positivity properties of Chern classes of auxiliary vector bundles. This approach allows one to derive concrete intersection inequalities and vanishing results that are difficult to access using purely abstract or algebraic methods.