A summary of works

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In [1], I determined the structure of the mod p homology ring of double loop spaces of Stiefel manifolds. To do this, I applied a notion called the Eilenberg-Moore spectral sequence as a main tool of the calculation and I showed two theorems in [2] on the Eilenberg-Moore spectral sequence of which I made use in order to calculate the spectral sequences.

Morava K-theory is a generalized homology theory which is a generalization of K-theory. [3] is my doctoral dissertation at The Johns Hopkins University and I determined the ring structure of Morava K-theory of double loop spaces of spheres.

Let p be a prime number. A topological space is said to be p-regular if the localization of X at p is homotopy equivalent to a product of p-localized spheres. J.P.Serre gave a necessary and sufficient condition for classical Lie groups to be p-regular and I tried extend this Serre's result to Stiefel manifolds. In the case that p is 2 or 3, I showed that a necessary condition for Stiefel manifolds to be p-regular obtained by examining the actions of the Steenrod on the cohomologies of Stiefel manifolds is also a sufficient condition. In the case that p is bigger than or equal to 5, for a complex Stiefel manifold $SU(n+k)/SU(n)$ and a quaternionic Stiefel manifolds $Sp(n+k)/Sp(n)$, I showed that the necessary condition is also sufficient except for finite number of pairs (n, k).

By applying complex cobordism theory, a spectral sequence called the Adams-Novikov spectral sequence is defined, which coverges to the stable homotopy group of spheres. Moreover, there is a spectral sequence which converges to the E_2-term of the Adams-Novikov spectral sequence and its E_2-term is given by the cohomology of an algebra called Morava stabilizer algebra. I determined the ring structure of the cohomology of Morava stabilizer algebra.

The notion of topological group is generalized homotopy theoretically to a notion "Hopf space" which is a monoid in the homotopy category of topological spaces. J. Harper constructed a Hopf space $K(p)$ for each odd prime number p which is a simply connected finite cell complex such that its integral homology has p-torsion. I described the Hurewicz homomorphism from the homotopy groups of $K(p)$ to the complex bordism homology groups of $K(p)$ in [6] by making full use of homotopy theoretical techniques.

By considering the notion of groupoid obtained from one dimensional formal group laws, I determined the structure of an algebraic object called "Hopf algebroid" in the category of graded commutative algebras associated with elliptic homology theory in [7].

I determined the ring structure of real K-cohomology of complex projective spaces in [8] and the ring structure of real K-homology of the infinite dimensional complex projective space in [9]. These results are somewhat experimental but these are motivated by a search for some relationships between real K-(co)homology and the Hopf algebroid associated with real K-theory.

The Steenrod algebra, which is the ring of operators on mod p cohomology groups, has a filtration defined from a notion called "excess". I generalized the notion of the Steenrod algebra by extracting the properties of this filtration of the Steenrod algebra in [10]. Moreover, I extended the notion of unstable modules over the Steenrod algebra to the notion of unstable modules over generalized Steenrod algebras and tried to reconstruct the theory of unstable modules. I also gave an example of a generalized Steenrod algebra whose quotient algebra by some ideal is the Steenrod algebra by embedding the group scheme represented by the dual Steenrod algebra into an infinite dimensional matrix group.

If a generalized homology E satisfies a certain condition, there is a Hopf algebroid associated with E and E-homology $E_*(X)$ of a space X has a structure of a comodule over the associated Hopf algebroid. Since a Hopf algebroid is nothing but a groupoid in the dual category of the category of graded commutative algebras, a comodule over a Hopf algebroid is considered as a "representation" of the groupoid obtained from a Hopf algebroid. In [11], I defined the notion of representations of general groupoids (more generally internal categories) in a category with finite limits by using the notion of fibered category and set a foundation to regard generalized homology theory by giving definitions of the pull-back of a representation by a morphism between internal categories and the notion of regular representations.

[12] is a joint work with Kishimoto. Here, we showed an inequality which estimates the difference of the increase of the topological complexity of a space by applying rational homotopy theory.

Let G be the group scheme represented by the dual Steenrod algebra over a prime field of characteristic p. I showed that G is isomorphic to a certain subgroup scheme of the automorphism group scheme of additive formal group law in [10]. Using this result, I defined a suitable decreasing filtration of G consisting of subgroup schemes and estimated the size of the n-th subgroup in the lower central series of G in [13]. I also showed the nilpotence of finite subgroup schemes of G and estimated the length of the lower central series of them. In addition to those, I constructed a sequence $G = G_0 \xrightarrow{\pi_0} G_1 \xrightarrow{\pi_1} \cdots \xrightarrow{\pi_{n-1}} G_n \xrightarrow{\pi_n} G_{n+1} \xrightarrow{\pi_{n+1}} \cdots$ of quotient morphisms of group schemes such that $\operatorname{Ker}\pi_n$ is a maximal abelian subgroup of G_n which is an "elementary abelian abelian p-group.