Research Plan

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I plan to pursue four lines of research.

1) The first concerns the smooth quandle defined by Ishikawa [1]. This is an analogue of Lie groups in group theory, namely a quandle endowed with a manifold structure and smooth operations. “The smooth quandle can be embedded into the conjugation quandle formed by an appropriate Lie group”[1] (Io: see [4] for reference). The author has shown the conjecture in the spherical quandle case, but it is not understood in other cases. I would like to investigate the other cases.

2) The second concerns the classification of finite/discrete subquandles of the quandle arising from symmetric spaces, and the extraction of the corresponding quandle cocycles. For researchers who hold the view that a quandle is a discretization of a symmetric space, this is expected to provide important concrete examples. Moreover, studying what symmetries these quandles possess and what groups act on them is seen as a path toward extracting quandle cocycles, which is demanded from the knot-theory side.

3) The third concerns Lie algebras and their representations. There is a framework called PC Lie algebras proposed by Sasano [3]. We aim to build new Lie algebras from Lie algebras and their representations, providing algebraic structures that encompass Kac–Moody Lie algebras. I consider PC Lie algebras to be a useful tool for studying smooth quandles. For example, there are examples of smooth quandles defined using the structure of Riemannian symmetric spaces, but how their quandle structure relates to root systems and Satake diagrams has not been explored at all. I think the theory of PC Lie algebras will be necessary to articulate and extend such investigations with concrete examples. Among the foundational theory, there remain unresolved problems (for instance, the construction of notions corresponding to real forms, explicit constructions within the PC Lie algebra framework for classical Lie algebras, and the correspondence between tensor-product representations and the resulting PC Lie algebras). I would like to approach these while considering applications to quandles.

4) The fourth concerns orbital decomposition. I participated in OCAMI’s joint-use project on “mathematical issues accompanying orbital decomposition.” The task is to study the sizes of double cosets of a certain algebraic group. Here, I have been writing programs to perform the computations needed to establish the theorems. For example, I have built a Python program to enumerate all matrices satisfying the conditions in concrete cases such as the double cosets of $GL(2n, \mathbb{F}_q)$ with $n = 2, 3, 4, 5, 6$. Going forward, I plan to deepen my understanding of the material and develop additional supporting programs.

References

[1] Katsumi Ishikawa. On the classification of smooth quandles. preprint.

[2] Takefumi Nosaka. *Quandles and topological pairs;Symmetry, knots, and cohomology.* SpringerBriefs in Mathematics. Springer, Singapore, 2017. Symmetry, knots, and cohomology.

[3] Nagatoshi Sasano. Contragredient lie algebras and lie algebras associated with a standard pentad. *Tsukuba Journal of Mathematics*, Vol. 42, No. 1, pp. 1–51, 2018.

[4] Kentaro Yonemura. *An embedding of a smooth quandle into a Lie group.* PhD thesis, Kyushu University doctor thesis, 2023.

[1] The quandle is defined by the right-conjugation action of the group. [2] references.