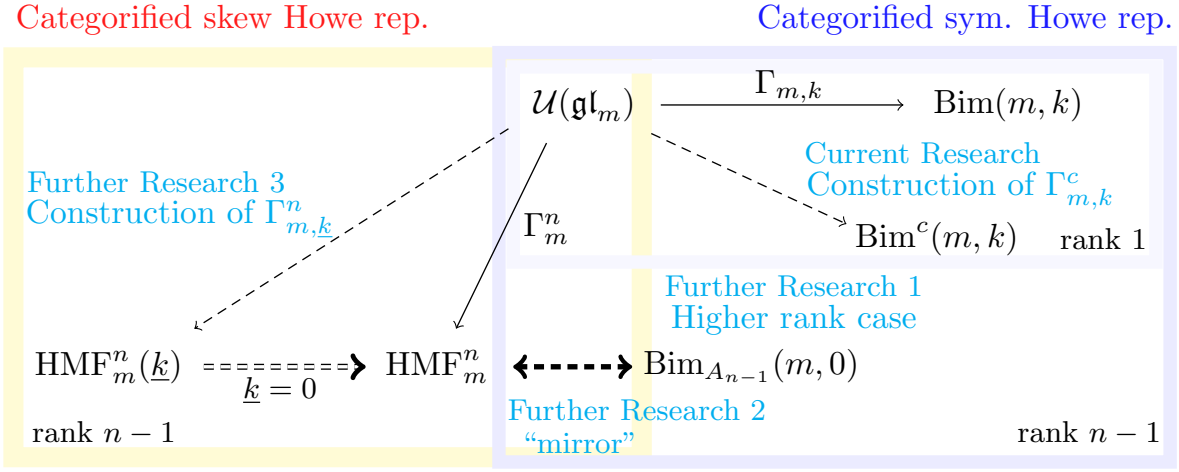

In the figure, solid arrows indicate results I have already obtained, while dashed arrows show research directions I plan to pursue.

Previously, I defined the general-type deformed Webster algebras $W^{\mathfrak{g}}$ as subalgebras of the Khovanov–Lauda–Rouquier algebra. Since the Khovanov–Lauda–Rouquier algebra carries a p-DG structure, $W^{\mathfrak{g}}$ naturally inherits a p-DG structure. Building on $W^{\mathfrak{g}}$, I will work on constructing homological link invariants that refine the quantum invariants associated with the quantum groups $U_q(\mathfrak{g})$ and their representations, and on developing homological invariants of three-dimensional manifolds.

Research Plan

This research will be carried out in accordance with the Grant-in-Aid for Scientific Research (KAKENHI), Project No. 22K03318.

In the 2026 academic year, I will advance my research based on discussions held during Khovanov's four-day visit to Japan in February 2025 and on discussions with Aaron Lauda and Joshua Sussan during my stay in August 2025.

(1) On the symmetric power $S^k(\mathbb{C}^n \otimes \mathbb{C}^m)$ there is a left action of $U_q(\mathfrak{sl}_n)$ and a right action of $U_q(\mathfrak{gl}_m)$ that commute. Hence we obtain the representation

$$\gamma^{\mathfrak{sl}_n}_m : U_q(\mathfrak{gl}_m) \to \bigoplus_{\sum_{\alpha=1}^m i_\alpha = k,\, \sum_{\alpha=1}^m j_\alpha = k} \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(S^{i_1} \otimes \cdots \otimes S^{i_m}, S^{j_1} \otimes \cdots \otimes S^{j_m}).$$

It is expected that this representation admits a categorification in the bimodule category of the deformed Webster algebra $W^{A_{n-1}}$. I will continue to pursue this problem during the coming year.

(2) Since the Khovanov–Lauda–Rouquier algebras carry a p-DG structure, the subalgebra $W^{\mathfrak{g}}$ naturally inherits a p-DG structure. Using this p-DG structure, I will study the categorification of representations of quantum groups at roots of unity.

(3) The above items (1) and (2) study categorifications of structures that arise from symmetric tensor products. I expect that analogous categorifications can be constructed for anti-symmetric tensor products using the category of matrix factorizations HMF.