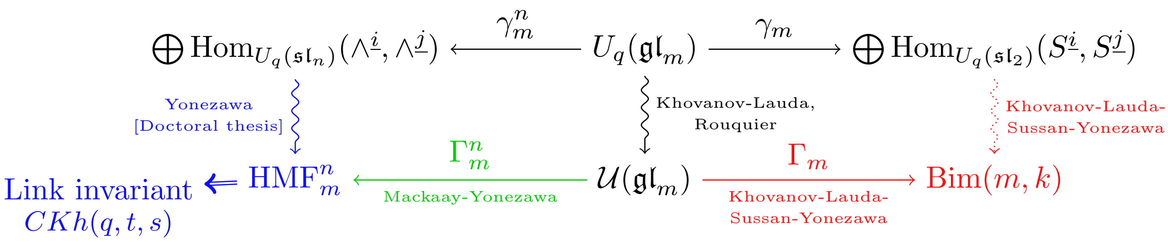
M. Khovanov introduced a homological link invariant that refines (categorifies) the Jones polynomial. The Jones polynomial is a quantum link invariant arising from the quantum group $U_q(\mathfrak{sl}_2)$ and its two-dimensional irreducible representation. Motivated by this, I have pursued the following question:

Can we construct homological link invariants that refine other quantum link invariants?

Quantum invariants defined from quantum groups at roots of unity can be extended to invariants of three-dimensional manifolds. Accordingly, I have also been investigating the following question:

Can we construct homological invariants of 3-manifolds?



(1) Summary of the paper "Quantum $(\mathfrak{sl}_n, \wedge V_n)$ link invariant and matrix factorizations": Khovanov and Rozansky constructed homological link invariants that categorify the quantum invariants associated with the quantum group $U_q(\mathfrak{sl}_n)$ and its n-dimensional irreducible representation. In this paper, we generalize the Khovanov–Rozansky theory and define a link invariant $CKh(q,t,s)$ that refines the invariant $CJ_n(q)$ arising from $U_q(\mathfrak{sl}_n)$ and its fundamental representations (the work highlighted in blue in the figure above). In particular, $CJ_n(q)$ is recovered by $CJ_n(q) = CKh(q,-1,1)$.

(2) Summary of the paper "$\mathfrak{sl}_N$-Web categories and categorified skew Howe duality": We constructed a functor $\Gamma_m^n : \mathcal{U}(\mathfrak{gl}_m) \to \mathrm{HMF}_m^n$, where $\mathcal{U}(\mathfrak{gl}_m)$ is a categorification of the quantum group $U_q(\mathfrak{gl}_m)$ and HMF_m^n denotes a category of matrix factorizations (highlighted in green in the figure above). Using this functor together with the braid group action on $\mathcal{U}(\mathfrak{gl}_m)$, we constructed homological link invariants that categorify the link invariants arising from $U_q(\mathfrak{sl}_n)$ and its fundamental representations.

(3) Summary of the paper "Braid group actions from categorical symmetric Howe duality on deformed Webster algebras": We introduced deformed Webster algebras $W(\mathbf{s},k)$ and constructed a functor

$$\Gamma_m : \mathcal{U}(\mathfrak{gl}_m) \longrightarrow \mathrm{Bim}(m,k),$$

where $\mathrm{Bim}(m,k)$ denotes the bimodule category of $W(\mathbf{s},k)$ (shown in orange in the figure). Using this functor, we constructed an action of the braid group on the bimodule category $\mathrm{Bim}(m,k)$.

(4) Summary of the paper "A braid group action on a p-DG homotopy category": Khovanov proposed categorifying structures at a root of unity by equipping categories with a p-DG structure. In this paper, we introduce a p-DG structure on the bimodule category $\mathrm{Bim}(m,k)$ and construct a braid group action on $\mathrm{Bim}(m,k)$ that is compatible with this p-DG structure.