Toshiya Yurikusa

【Future Research Plan】

Building on my previous results on the denseness of g-fans and the equivalence of finiteness and tameness conditions for Jacobian algebras, my future research aims to develop a unified theory that integrates finiteness, tameness, and stability conditions. In particular, I plan to focus on the following three research directions.

1. Development of a unified theory of g-tame, E-tame, and representation-tame algebras

Based on the equivalences established in Haerizadeh–Yurikusa (2025) for finite-dimensional Jacobian algebras, I aim to extend this framework to infinite-dimensional Jacobian algebras and more general infinite-dimensional algebras. The main objectives are as follows:

(a) Analyze the behavior of g-fans and E-invariants of truncations, and clarify criteria for finiteness in infinite-dimensional Jacobian algebras.

(b) Use τ-regular components of representation moduli spaces to organize the relationships among g-vectors, E-invariants, and stability conditions, and propose unified definitions of finiteness and tameness.

Through this framework, I also plan to approach the τ-regular Brauer–Thrall II' conjecture and the stable Brauer–Thrall II' conjecture.

2. Classification of cluster algebras with dense g-fans (exceptional types)

While denseness of g-fans has been established for surface types (Yurikusa, 2020) and tame types (Plamondon–Yurikusa, 2023), exceptional types such as X_6 and $F_4^{(*,+)}$ remain unresolved. In this project, I plan to:

(a) Analyze wall structures of g-fans using Cambrian fans (Reading–Speyer, 2018) and extended affine root systems (Saito, 1985), and study the limiting behavior of g-vectors.

(b) Compare exceptional types with surface and tame types from a geometric viewpoint, and complete a classification of cluster algebras whose g-fans are dense.

3. Connections with stability conditions, scattering diagrams, and DT theory

Cluster algebras are deeply connected to stability conditions, wall-crossing phenomena, and Donaldson–Thomas theory. In particular, g-fans naturally correspond to parts of stability spaces and scattering diagrams. I plan to:

(a) Investigate the limiting behavior of g-vectors and analyze regions of stability spaces and scattering diagrams that are not captured by g-fans.

(b) Clarify the relationship between stability conditions and DT theory through the geometry of g-fans.

Through these projects, I aim to construct a unified framework that integrates cluster structures and stability spaces.