

## (2) Summary of Research Achievements and Future Research Plan

### 【Summary of Research Achievements】

My research focuses on  $g$ -vectors and  $g$ -fans (or  $g$ -vector fans), which appear naturally in both cluster algebras and the representation theory of finite-dimensional algebras. In cluster algebras,  $g$ -vectors are closely related to wall structures of scattering diagrams and the construction of canonical bases, while in representation theory they are connected to torsion pairs, stability conditions, and  $\tau$ -tilting theory. By regarding  $g$ -vectors as a common language between these two fields, my aim is to understand their structures in a unified and geometric manner, with particular emphasis on the relationship between geometric properties of  $g$ -fans and various finiteness conditions.

#### ■ Main Research Results

##### 1. Denseness of $g$ -fans for surface-type cluster algebras (Yurikusa, 2020)

I clarified the correspondence between  $g$ -vectors of cluster algebras associated with surfaces and shear coordinates of laminations. Using geometric techniques based on Dehn twists, I proved that the  $g$ -fan is dense. As an application, I showed that the mutation graph of support  $\tau$ -tilting pairs for the associated Jacobian algebra is connected.

##### 2. Denseness of $g$ -fans for tame finite-dimensional algebras (Plamondon–Y., 2023)

I provided a representation-theoretic (categorical) interpretation of the geometric ideas used in the surface-type case and proved that the  $g$ -fan is dense for all representation-tame finite-dimensional algebras, including string algebras and Brauer graph algebras. Concretely, I replaced the decomposition of laminations by the canonical decomposition of  $g$ -vectors, and Dehn twists by an operation called a cylinder in triangulated categories, which allowed me to analyze the limiting behavior of  $g$ -vectors. This result introduces a new viewpoint on the classical tame/wild dichotomy via the geometry of  $g$ -fans and strengthens the connection between representation theory and cluster-algebraic geometry.

##### 3. Unification of finiteness and tameness for Jacobian algebras (Haerizadeh–Y., 2025)

For finite-dimensional Jacobian algebras, we proved that the following conditions are equivalent:  $g$ -finiteness,  $E$ -finiteness,  $\tau$ -tilting finiteness, and representation-finiteness. In particular, the equivalence between  $E$ -finiteness and  $\tau$ -tilting finiteness resolves a conjecture posed by Demonet (2017) for general finite-dimensional algebras, within the class of Jacobian algebras. We also established equivalences among  $g$ -tameness,  $E$ -tameness, and representation-tameness for cluster-tilted algebras, leading to a unified understanding of finiteness and tameness conditions.

These results demonstrate that  $g$ -vectors and  $E$ -invariants provide a powerful common framework for understanding finiteness and tameness phenomena in the representation theory of finite-dimensional algebras.