Classifying the surface-knot modules

Akio Kawauchi

Osaka Central Advanced Mathematical Institute, Osaka Metropolitan University

Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan

kawauchi@omu.ac.jp

ABSTRACT

The kth module of a surface-knot of a genus g in the 4-sphere is the kth integral homology module of the infinite cyclic covering of the surface-knot complement. The reduced first module is the quotient module of the first module by the finite sub-module defining the torsion linking. It is shown that the reduced first module for every genus g is characterized in terms of properties of a finitely generated module. As a by-product, a concrete example of the fundamental group of a surface-knot of genus g which is not the fundamental group of any surface-knot of genus $g-1$ is given for every $g>0$. The torsion part and the torsion-free part of the second module are determined by the reduced first module and the genus-class on the reduced first module. The third module vanishes. The concept of an exact leaf of a surface-knot is introduced, whose linking is an orthogonal sum of the torsion linking and a hyperbolic linking.

Keywords: Surface-knot, Surface-basis, Genus-class, Exact leaf.
Mathematics Subject Classification 2010: Primary 57N13; Secondary 57Q45

1. Introduction

A *surface-knot* is a closed (connected oriented) surface F with genus $g(\geq 0)$ smoothly embedded in the 4-sphere S^4. Let $E = E(F) = \mathrm{cl}(S^4 \setminus N(F))$ be the exterior of a surface-knot F, where $N(F) = F \times D^2$ is a normal disk bundle of F in S^4, where the section $F \times 1$ of the circle bundle $\partial N(F) = \partial E = F \times S^1$ of F is chosen so that the natural homomorphism $H_1(F \times 1; Z) \to H_1(E; Z) = Z$ is the zero map. Let proj : $\tilde{E} \to E$ be the infinite cyclic connected covering belonging to the kernel of the canonical epimorphism $\pi_1(E, x_0) \to H_1(E; Z) = Z$. Then the section $F \times 1$ of ∂E lifts to the section $F \times 0$ of $\partial E = F \times \boldsymbol{R}$. Let $\Lambda = Z[Z] = Z[t, t^{-1}]$ be the integral group ring of the infinite cyclic covering transformation group $< t >$ of $\tilde{E}$ with generator t identified with the meridian generator of F in $H_1(E; Z) = Z$. The *kth surface-knot module* (or simply the *kth module* of a surface-knot F in S^4 is the kth integral homology group $A_k(F) = H_k(\tilde{E}; Z)$ considered as a finitely generated Λ-module. For a finitely generated Λ-module H, let TH be the Λ-torsion part of H and $BH = H/TH$, the Λ-torsion-free part of H. Let DH be the Λ-submodule of TH consisting of every element x with $f_i(t)x = 0\,(i = 1, 2, \ldots, s)$ for a coprime element system $f_i(t) \in \Lambda\,(i = 1, 2, \ldots, s)$, which is the maximal finite Λ-submodule of TH, and $T_D H = TH/DH$. Let $E^q(H) = Ext^q_\Lambda(H, \Lambda)$ be the qth extension cohomology Λ-module of H. Since Λ is a Noetherian ring of global dimension 2, $E^q(H)$ is always finitely generated and $E^q(H) = 0\,(q \geq 3)$. In particular, $E^0(H) = \hom_\Lambda(H, \Lambda)$ is a free Λ-module, whose Λ-rank is defined to be the Λ-*rank* of H. It is a standard fact that there is a natural short exact sequence

$$0 \to E^1(BH) \to E^1(H) \to E^1(TH) \to 0,$$

where $E^1(BH)$ is a finite Λ-module and $E^1(H)$ is a finitely generated torsion Λ-module with

$$E^1(BH) \cong DE^1(H), \quad T_D E^1(H) = \hom_\Lambda(TH, Q(\Lambda)/\Lambda) = \hom_\Lambda(T_D H, Q(\Lambda)/\Lambda)$$

for the quotient field $Q(\Lambda)$ of Λ and $E^1E^1(H) = E^1E^1(T_D H) = T_D H$. The Λ-module $E^2(H)$ is a finite Λ-module with $E^2(H) = \hom_Z(DH, Q/Z)$ and $E^2E^2(H) = E^2E^2(DH) = DH$. It is also a standard fact that there is a natural short exact sequence

$$0 \to BH \to E^0E^0(BH) \to E^2E^1(BH) \to 0.$$

A $(t-1)$-*divisible* Λ-module is a finitely generated Λ-module H such that the multiplication $t - 1 : H \to H$ is a Λ-isomorphism. Then every Λ-submodule and every quotient Λ-module of H are torsion $(t-1)$-divisible Λ-modules and DH is a finite Λ-module. See [7, 17, 18] for these properties of $E^q(H)$.

An r-*weight* of a finite Λ-module D is a Λ-epimorphism $\omega : \Lambda^r \to D$. Two r-weights ω and ω' of D are *equivalent* if there are Λ-isomorphisms $f_\Lambda : \Lambda^r \to \Lambda^r$ and $f_D : D \to D$ such that $\omega' = f_D \omega f_\Lambda^{-1}$. An r-*class* on D is the equivalence class $[\omega]$ of an r-weight ω of D. For every r, there are only finitely many r-classes on D, where if there is no Λ-epimorphism $\Lambda^r \to D$, then we understand that D has the *empty r-class* $[\emptyset]$. For every non-empty r-class $[\omega]$ on D, then there is a unique (up to Λ-isomorphisms) torsion-free Λ-module B such that the natural Λ-epimorphism $E^0E^0(B) \to E^2E^1(B)$ is equivalent to ω. (see Lemma 4.1).

Elementary computations show that $A_k(F) = 0$ except for $0 \leq k \leq 3$, and $A_0(F) = Z$ (regarded as a Λ-module with trivial t-action). By the zeroth duality of [7], there is a non-degenerate Λ-Hermitian form

$$S : E^0E^0(BA_2(F)) \times E^0E^0(BA_2(F)) \to \Lambda$$

as an invariant of a surface-knot F in S^4 with the identities

$$f(t)S(x, x') = S(f(t^{-1})x, x') = S(x, f(t)x') \quad (x, x' \in BA_2(F),\ f(t) \in \Lambda)$$

extending the non-degenerate Λ-intersection form

$$S^B : BA_2(F) \times BA_2(F) \to \Lambda$$

defined by

$$S^B(x, x') = \mathrm{Int}_\Lambda(x, x') = \sum_{i=-\infty}^{+\infty} \mathrm{Int}(x, t^i x')t^{-i} \in \Lambda.$$

By the second duality of [7], the torsion linking (that is a t-isometric symmetric bilinear non-singular pairing)

$$\ell_F : \Theta(F) \times \Theta(F) \to Q/Z$$

on a finite Λ-module $\Theta(F)$ in $DA_1(F)$ is defined as an invariant of a surface-knot F in S^4. The *reduced first module* of F in S^4 is the quotient Λ-module $R_1(F) = A_1(F)/\Theta(F)$ of the first module $A_1(F)$, which is an invariant of a surface-knot F in S^4. Let $e(H)$ denote the minimal number of Λ-generators of H. The following theorem is the main result of this paper.

Theorem 1.1. The kth surface-knot modules $A_k(F)\,(1 \le k \le 3)$ of every surface-knot F of genus $g > 0$ in S^4 have the following properties.

(1) A Λ-module H is Λ-isomorphic to the reduced first module $R_1(F)$ of a surface-knot F in S^4 of genus $g\,(\ge 0)$ if and only if H is a $(t-1)$-divisible finitely generated Λ-module with inequality $e(E^2(H)) \le g$.

(2) Every surface-knot F in S^4 of genus g defines a g-class invariant $[\omega_F]$ on the finite Λ-module $E^2(R_1(F))$ so that the reduced first module $R_1(F)$ and the g-class $[\omega_F]$ determine the Λ-modules $TA_2(F)$ and $BA_2(F)$ up to Λ-isomorphisms. In particular, there are t-anti Λ-isomorphisms

$$E^1(R_1(F)) \cong TA_2(F), \quad E^2(R_1(F)) \cong E^2E^1(BA_2(F)).$$

(3) There is a direct sum splitting $BA_2(F) = X_F \oplus Y_F$ with Y_F a free Λ-module of rank g such that the Λ-Hermitian form

$$S : E^0E^0(BA_2(F)) \times E^0E^0(BA_2(F)) \to \Lambda$$

is given by

$$S(x_i, x_j) = S(y_i, y_j) = 0, \quad S(x_i, y_j) = (t-1)\delta_{ij} \quad (i, j = 1, 2, \ldots, g)$$

for a Λ-basis $x_i, y_i\,(i = 1, 2, \ldots, g)$ of $E^0E^0(BA_2(F)) = E^0E^0(X_F) \oplus Y_F$ with $x_i\,(i = 1, 2, \ldots, g)$ a Λ-basis of $E^0E^0(X_F)$ and $y_i\,(i = 1, 2, \ldots, g)$ a Λ-basis of Y_F.

(4) $A_3(F) = 0$.

The g-class $[\omega_F]$ on the finite Λ-module $E^2(R_1(F))$ is called the *genus-class invariant* of a surface-knot F. The weaker inequality $e(E^2(R_1(F))) \leq 2g$ has been earlier obtained and applied to surface-knot theory (see [8, p.192]). If F is an S^2-knot K in S^4, then $e(E^2(R_1(F))) = 0$, that is, $R_1(K)$ is a Z-torsion-free Λ-module, which is also the result of Farber-Levine pairing of an S^2-knot in S^4 ([1, 18]). This weaker inequality and the symmetric property of $\Theta(F)$ that $\Theta(F)$ admits a t-anti automorphism are applied to know implicitly the properness of the sequence

$$\boldsymbol{G}(0) \subset \boldsymbol{G}(1) \subset \boldsymbol{G}(2) \subset \cdots \subset \boldsymbol{G}(g) \subset \ldots$$

where $\boldsymbol{G}(g)$ denotes the set of the fundamental groups of surface-knots of genus g (see [12]) and the properness of the sequence

$$\boldsymbol{A}(0) \subset \boldsymbol{A}(1) \subset \boldsymbol{A}(2) \subset \cdots \subset \boldsymbol{A}(g) \subset \ldots$$

where $\boldsymbol{A}(g)$ denotes the set of the first modules of surface-knots of genus g (see [13]). By Theorem 1.1 (1) and the symmetric property of $\Theta(F)$, the properness of these sequences can be shown with explicit examples as follows.

Corollary 1.2. For every prime $p \geq 5$, consider the finite Λ-module $D = \Lambda/(p, 2t-1)$ and the ribbon presented group

$$\pi = < x, y |\, y = (x^{-1}y)x(y^{-1}x),\ y = (xy^{-1})^p y (yx^{-1})^p > .$$

Then there is a ribbon torus-knot T in S^4 with fundamental group $\pi_1(S^4 \setminus T, x_0) = \pi$ and $A_1(T) = D$. For every integer $g \geq 1$, let T_g be the g-fold connected sum of T in S^4 which is a ribbon surface-knot of genus g. Then the fundamental group $\pi_1(S^4 \setminus T_g, x_0)$ which has a ribbon presentation

$$< x, y_1, y_2, \ldots, y_g |\, y_i = (x^{-1}y_i)x(y_i^{-1}x),\ y_i = (xy_i^{-1})^p y_i (y_i x^{-1})^p,\ i = 1, 2, \ldots, g >$$

belongs to $\boldsymbol{G}(g) \setminus \boldsymbol{G}(g-1)$ and the first module $A_1(T_g)$ of T_g in S^4 belongs to $\boldsymbol{A}(g) \setminus \boldsymbol{A}(g-1)$.

A basic idea of the proof of Theorem 1.1 is to construct a surface-basis for every surface-knot F of genus g in S^4 to apply the 3 dualities in [7] which is described from now. A *loop basis* for a closed oriented surface F of genus $g > 0$ is a system of simple loops $\alpha_i, \alpha'_i\,(i = 1, 2, \dots, g)$ in F such that

$$\alpha_i \cap \alpha_j = \alpha_i \cap \alpha'_j = \emptyset\,(i \neq j) \quad \text{and} \quad \alpha_i \cap \alpha'_i = p_i, \text{a point.}$$

A loop basis $\alpha_i, \alpha'_i\,(i = 1, 2, \dots, g)$ of a surface-knot F is *spin* if $q([\alpha_i]_2) = q([\alpha'_i]_2) = 0$ for all i with respect to the quadratic function $q : H_1(F; Z_2) \to Z_2$ associated with the surface-knot F in S^4. By [3], there is a spin loop basis for every surface-knot F in S^4.

Definition. A *surface-basis* of a surface-knot F in S^4 of genus $g > 0$ is a system of (compact connected oriented) surfaces $D_i, D'_i\,(i = 1, 2, \dots, g)$ smoothly embedded in S^4 such that

(1) $D_i \cap F = \partial D_i = \alpha_i$ and $D'_i \cap F = \partial D'_i = \alpha'_i$, $(i = 1, 2, \dots, g)$) for a spin loop basis $\alpha_i, \alpha'_i\,(i = 1, 2, \dots, g)$ of F,

(2) $D_i \cap D_j = D'_i \cap D'_j = D_i \cap D'_j = \emptyset\,(i \neq j)$, and the self Z-intersection numbers $\mathrm{Int}(D_i, D_i) = \mathrm{Int}(D'_i, D'_i) = 0$ with respect to the surface framing of F for all i, and

(3) the natural homomorphisms $H_1(D_i \setminus \alpha_i; Z) \to H_1(S^4 \setminus F; Z)$ and $H_1(D'_i \setminus \alpha'_i; Z) \to H_1(S^4 \setminus F; Z)$ are the zero maps for all i.

In this definition, note that no information on the intersection between the interior $\mathrm{Int}D_i$ of D_i and the interior $\mathrm{Int}D'_i$ of D'_i is given for every i. and the interchange between some surfaces in $D_i\,(i = 1, 2, \dots, g)$ and the corresponding surfaces in $D'_i\,(i = 1, 2, \dots, g)$ makes a surface-basis for F in S^4. The following theorem is basically important in this paper, which is shown in Section 2.

Theorem 1.3. For every spin loop system $\alpha_i, \alpha'_i\,(i = 1, 2, \dots, g)$ of a surface-knot F of genus g in S^4, there is a surface-basis $D_i, D'_i\,(i = 1, 2, \dots, g)$ for a surface-knot F in S^4 with $\partial D_i = \alpha_i, \partial D'_i = \alpha'_i\,(i = 1, 2, \dots, g)$.

A *leaf* (or in other words, a *Seifert hypersurface*) of a surface-knot F in S^4 is a compact connected oriented 3-manifold V_F (smoothly embedded) in S^4 with $\partial V_F = F$, which is always exists (see [2], [16, II]). A leaf V_F is also considered as a proper 3-submanifold of E with $\partial V_F = F \times 1 \subset F \times S^1 = \partial N(F)$. Then the homology class $[V_F] \in H_3(\tilde{E}, \partial\tilde{E}; Z)$ is just the *fundamental class* of the covering $\mathrm{proj} : \tilde{E} \to E$ (see

[9]). A leaf V_F of F in E is *exact* if the sequence

$$0 \to \mathrm{Tor}H_2(\tilde{E}, \tilde{V}_F; Z) \to \mathrm{Tor}H_1(\tilde{V}_F; Z) \to \mathrm{Tor}H_1(\tilde{E}; Z)$$

is exact. This notion is a variation of a closed exact leaf on a closed oriented 4-manifold with infinite cyclic first homology group in [10].

Theorem 1.4. For every surface-basis $D_i, D'_i\,(i = 1, 2, \ldots, g)$ of every surface-knot F of genus g in S^4, there is an exact leaf V_F containing the half surface-basis $D_i\,(i = 1, 2, \ldots, g)$ as proper surfaces.

A *hyperbolic linking* is a linking (i.e., non-singular symmetric bilinear form) $\ell : G^2 \times G^2 \to Q/Z$ on the direct double G^2 of a finite abelian group G such that $\ell(x, x) = 0$ for all $x \in G$ (see [15]). The following corollary is a combination result of Theorem 1.4 and an earlier result on a closed exact leaf in [11].

Corollary 1.5. The torsion linking $\ell_F : \Theta(F) \times \Theta(F) \to Q/Z$ of every surface-knot F in S^4 is an orthogonal summand of the linking $\ell_V : \mathrm{Tor}H_1(V_F; Z) \times \mathrm{Tor}H_1(V_F; Z) \to Q/Z$ for every exact leaf V_F containing the half surface-basis $D_i\,(i = 1, 2, \ldots, g)$ of every surface-basis $D_i, D'_i\,(i = 1, 2, \ldots, g)$ as proper surfaces, which is a non-singular linking and whose complement linking is a hyperbolic linking.

In Section 2, a surface-basis for every surface-knot is constructed. In Section 3, the surface-knot manifold M which is a closed spin 4-manifold with $H_1(M; Z) \cong Z$ obtained from S^4 by a surgery along the surface-knot F is considered to apply the 3 dualities of [7] to the integral infinite cyclic covering homology $H_*(\tilde{M}; Z)$ where a surface-basis of a surface-knot is used. In Section 4, the proofs of Theorems 1.1 and Corollary 1.2 are given. In Section 5, Theorem 1.4 and Corollary 1.5 are shown by using a closed exact leaf of the surface-knot manifold M is discussed in [10, 11].

2. A surface-basis of a surface-knot

A *surface-basis in the weak sense* for a surface-knot F of genus $g > 0$ in S^4 is a surface-basis for F that does not impose the condition (3). Namely, there are (compact connected oriented) surfaces $D_i, D'_i\,(i = 1, 2, \ldots, g)$ smoothly embedded in S^4 such that

(1) $D_i \cap F = \partial D_i = \alpha_i$ and $D'_i \cap F = \partial D'_i = \alpha'_i$, $(i = 1, 2, \ldots, g)$ for any given spin loop basis $\alpha_i, \alpha'_i\,(i = 1, 2, \ldots, g)$ of F, and

(2) the intersection numbers $\mathrm{Int}(D_i, D_j) = \mathrm{Int}(D'_i, D'_j) = \mathrm{Int}(D_i, D'_j) = \mathrm{Int}(D'_i, D_j) = 0\,(i \neq j)$, and the self Z-intersection numbers $\mathrm{Int}(D_i, D_i) = \mathrm{Int}(D'_i, D'_i) = 0$ with respect to the surface framing of F are 0 for all i in S^4.

A surface-basis in the weak sense is constructed in [3] for every surface-knot F in S^4 with any given spin loop basis $\alpha_i, \alpha'_i\,(i = 1, 2, \ldots, g)$. To be precise, the condition that $\mathrm{Int}(D_i, D'_j) = 0\,(i \neq j)$ is omitted in [3], but it is shown as well. For a spin loop basis $\alpha_i, \alpha'_i\,(i = 1, 2, \ldots, g)$ in $F \times 1 \subset \partial E$, let $D_i, D'_i\,(i = 1, 2, \ldots, g)$ be a surface-basis in the weak sense in E with $\partial D_i = \alpha_i$, $\partial D'_i = \alpha'_i$, $(i = 1, 2, \ldots, g)$. Let $T_i = \ell_i \times S^1, T'_i = \ell'_i \times S^1\,(i = 1, 2, \ldots, g)$ be the tori in ∂E. Let $a(D_i) = T_i \cup D_i$ and $a(D'_i) = T'_i \cup D_i$ be be the 2-cycles in E homologous to T_i and T'_i, respectively. The elements $[a(D_i)], [a(D'_i)]\,(i = 1, 2, \ldots, g)$ form a basis of $H_2(E; Z)$ whose dual basis of $H_2(E, \partial E; Z)$ with respect to the non-singular intersection form

$$\mathrm{Int}_\partial : H_2(E; Z) \times H_2(E, \partial E; Z) \to Z$$

are given by the homology classes $[D'_i], [D_i]\,(i = 1, 2, \ldots, g)$. For the homological argument on the infinite cyclic covering $\tilde{E}$ of the exterior E of a surface-knot F of genus g, the following facts will be used throughout the paper:

Lemma 2.1. The exterior E of a surface-knot F of genus g has the following homological properties.
(1) $A_1(F)$ and $H_1(\tilde{E}, \partial\tilde{E}; Z)$ are $(t-1)$-divisible and there is a natural Λ-isomorphism $A_1(F) \cong H_1(\tilde{E}, \partial\tilde{E}; Z)$.

(2) $DA_2(F) = DH_2(\tilde{E}, \partial\tilde{E}; Z) = 0$.

(3) $TA_2(F)$ and $TH_2(\tilde{E}, \partial\tilde{E}; Z)$ are $(t-1)$-divisible, so that there is a natural Λ-isomorphism $TA_2(F) \cong TH_2(\tilde{E}, \partial\tilde{E}; Z)$ and there is a natural short exact sequence

$$0 \to BA_2(F) \stackrel{j_*}{\to} BH_2(\tilde{E}, \partial\tilde{E}; Z) \stackrel{\partial_*}{\to} H_1(\partial\tilde{E}; Z) \to 0$$

with $H_1(\partial\tilde{E}; Z) \cong Z^{2g}$.

(4) $E^1(BA_2(F))$ and $E^1(BH_2(\tilde{E}, \partial\tilde{E}; Z))$ are $(t-1)$-divisible and there is a natural Λ-isomorphism $E^1(BH_2(\tilde{E}, \partial\tilde{E}; Z)) \cong E^1(BA_2(F))$.

Technically, the following observation is useful (whose proof is direct and omitted).

Observation 2.2. In an exact sequence

$$H_0 \to H_1 \stackrel{\varphi}{\to} H_2 \to H_3$$

of finitely generated Λ-modules H_i $(0 \leq i \leq 3)$ and Λ-homomorphisms, if $(t-1)H_0 = (t-1)H_3 = 0$ and H_1, H_2 are $(t-1)$-divisible, then the Λ-homomorphism $\varphi : H_1 \to H_2$ is a Λ-isomorphism.

Proof of Lemma 2.1. Since $H_1(E;Z) \cong Z$, the Wang exact sequence shows that $t-1 : A_1(F) \to A_1(F)$ is an isomorphism, showing that $A_1(F)$ is $(t-1)$-divisible. This fact and the homology exact sequence of the pair $(\tilde{E}, \tilde{\partial E})$ shows that $TA_2(F) \cong TH_2(\tilde{E}, \partial\tilde{E}; Z)$, showing (1). By the second duality of [7], there are t-anti epimorphisms

$$\theta : DA_2(F) \to E^1(BH_1(\tilde{E}, \partial\tilde{E}; Z)) = 0, \quad \theta : DH_2(\tilde{E}, \partial\tilde{E}; Z) \to E^1(BA_1(F)) = 0$$

whose kernels $DA_2(F)^\theta = DA_2(F)$, $DH_2(\tilde{E}, \partial\tilde{E}; Z)^\theta = DH_2(\tilde{E}, \partial\tilde{E}; Z)$ are t-anti Λ-isomorphic to $\hom_Z(DH_0(\tilde{E}, \partial\tilde{E}; Z)^\theta, Q/Z) = 0$ and $\hom_Z(DA_0(F)^\theta, Q/Z) = 0$, for $DA_0(F) = DH_0(\tilde{E}, \partial\tilde{E}; Z) = 0$. Hence $DA_2(F) = DH_2(\tilde{E}, \partial\tilde{E}; Z) = 0$, showing (2). Then by the second duality of [7], $TA_2(F)$and $TH_2(\tilde{E}, \partial\tilde{E}; Z)$ are t-anti Λ-isomorphic to $E^1(T_D H_1(\tilde{E}, \partial\tilde{E}; Z)) = E^1(H_1(\tilde{E}, \partial\tilde{E}; Z))$ and $E^1(T_D A_1(F)) = E^1(A_1(F))$ which are $(t-1)$-divisible, respectively. This $(t-1)$-divisibility and the homology exact sequence of the pair $(\tilde{E}, \partial\tilde{E})$ show that the natural homomorphism $TA_2(F) \to TH_2(\tilde{E}, \partial\tilde{E}; Z)$ is a Λ-isomorphism, so that there is a natural short exact sequence

$$(*) \qquad 0 \to BA_2(F) \xrightarrow{j_*} BH_2(\tilde{E}, \partial\tilde{E}; Z) \xrightarrow{\partial_*} H_1(\partial\tilde{E}; Z) \to 0,$$

showing (3). By the second duality of [7], there are t-anti Λ-eimorphisms

$$\theta : DA_1(F) \to E^1(BH_2(\tilde{E}, \partial\tilde{E}; Z)),\ \theta : DH_1(\tilde{E}, \partial\tilde{E}; Z) \to E^1(BA_2(F)).$$

Since $DA_1(F), DH_1(\tilde{E}, \partial\tilde{E}; Z)$ are $(t-1)$-divisible by (1), it is seen that a natural Λ-homomorphism $E^1(BH_2(\tilde{E}, \partial\tilde{E}; Z)) \to E^1(BA_2(F))$ is a Λ-isomorphism by applying the extension cohomology to the short exact sequence $(*)$, showing (4). This completes the proof of Lemma 2.1.

By the zeroth duality of [7], the non-singular Λ-form

$$S_\partial : E^0E^0(BA_2(F)) \times E^0E^0(BH_2(\tilde{E}, \partial\tilde{E}; Z)) \to \Lambda$$

is given by extending the non-degenerate Λ-intersection form the non-degenerate Λ-Hermitian form

$$S_\partial^B : BA_2(F) \times BH_2(\tilde{E}, \partial\tilde{E}; Z) \to \Lambda,$$

defined by $S_\partial^B(x, x') = \mathrm{Int}_\Lambda(x, x') = \sum_{i=-\infty}^{+\infty} \mathrm{Int}(x, t^i x')t^{-i} \in \Lambda$ for $x \in BA_2(F)$, $x' \in BH_2(\tilde{E}, \partial\tilde{E}; Z)$.

Let Λ^+ be the subring of the quotient field $Q(\Lambda)$ of Λ generated by the products $u(t)^{-1}f(t)$ of any elements $u(t), f(t) \in \Lambda$ with $u(1) = \pm 1$. Note that the ring Λ^+ admits a t-anti automorphism. For a finitely generated Λ-module H, let $H^+ = H \otimes_\Lambda \Lambda^+$. It is a standard fact that *for every* $(t-1)$*-divisible finitely generated* Λ*-module* H*, there is an element* $u(t) \in \Lambda$ *such that* $u(1) = \pm 1$ *and* $u(t)H = 0$. (In fact, $H = TH$. Since $T_D H$ has projective dimension 1, there is a short exact sequence $0 \to \Lambda^m \overset{P(t)}{\to} \Lambda^m \to T_D H \to 0$, where $P(t)$ denotes a presentation Λ-matrix. Then $u_1(t) = \det P(t)$ has $u_1(t) T_D H = 0$. Since $T_D H$ is (t-1)-divisible, $P(1)$ is a unimodular matrix and $u_1(1) = \det P(1) = \pm 1$. On the other hand, some iteration $(t-1)^{m'}$ of $t-1$ acts on the finite Λ-module DH as the identity. Then $u_2(t) = 1-(t-1)^{m'}$ has $u_2(1) = 1$ and $u_2(t)DH = 0$. The product $u(t) = u_1(t)u_2(t)$ has $u(1) = \pm 1$ and $u(t)H = 0$, as desired.) Since $A_1(F) \cong H_1(\tilde{E}, \partial\tilde{E}; Z)$ is $(t-1)$-divisible, the second duality of [7] implies that $E^2E^1(BA_2(F))$ and $E^2E^1(BH_2(\tilde{E}, \partial\tilde{E}; Z))$ are $(t-1)$-divisible, so that $BA_2(F)^+ = E^0E^0(BA_2(F))^+$ and $BH_2(\tilde{E}, \partial\tilde{E}; Z)^+ = E^0E^0(BH_2(\tilde{E}, \partial\tilde{E}; Z))^+$ are free Λ^+-modules, and the non-degenerate Λ-Hermitian form S^B_∂ induces a *non-singular* Λ^+-Hermitian form

$$S^+_\partial = \mathrm{Int}_{\Lambda^+} : BA_2(F)^+ \times BH_2(\tilde{E}, \partial\tilde{E}; Z)^+ \to \Lambda^+$$

by defining

$$\mathrm{Int}_{\Lambda^+}(x, x') = u(t^{-1})^{-1}u'(t)^{-1}\mathrm{Int}_\Lambda(u(t)x, u'(t)x')$$

for $x \in BA_2(F)^+, x' \in BH_2(\tilde{E}, \partial\tilde{E}; Z)^+$ and $u(t), u'(t) \in \Lambda$ such that $u(1) = u'(1) = 1$ and $u(t)x \in BA_2(F), u'(t)x' \in BH_2(\tilde{E}, \partial\tilde{E}; Z)$. Similarly, the non-degenerate Λ-Hermitian form $S^B : BA_2(F) \times BA_2(F) \to \Lambda$ induces a non-degenerate Λ^+-Hermitian form

$$S^+ = \mathrm{Int}_{\Lambda^+} : BA_2(F)^+ \times BA_2(F)^+ \to \Lambda^+.$$

Note that there is a natural short exact sequence

$$0 \to BA_2(F)^+ \overset{i_*}{\to} BH_2(\tilde{E}, \partial\tilde{E}; Z)^+ \overset{\partial_*}{\to} H_1(\partial\tilde{E}; Z) \to 0$$

and $H_1(\partial\tilde{E}; Z) = Z^{2g}$ with the Z-basis represented by the spin loop basis $\alpha_i \times 0, \alpha'_i \times 0\,(i = 1, 2, \dots, g)$ of $F \times 0 \subset F \times \boldsymbol{R} = \partial\tilde{E}$. Note that

$$S^+(x, x') = S^+_\partial(x, i_*(x'))$$

for all $x, x' \in BA_2(F)^+$. A *well-defined pair* of relative 2-cycles in $BH_2(\tilde{E}, \partial\tilde{E}; Z)^+$ is a pair (c, c') of relative 2-cycles c, c' in $BH_2(\tilde{E}, \partial\tilde{E}; Z)^+$ such that the boundary 1-cycle pair $(\partial c, \partial c')$ is any pair of $\pm\alpha_i \times 0, \pm\alpha'_i \times 0\,(i = 1, 2, \dots, g)$ except for the unordered pair of $\pm\alpha_i \times 0$ and $\pm\alpha'_i \times 0$ for every i. For every well-defined pair (c, c'), the Λ^+-intersection number $\mathrm{Int}_{\Lambda^+}(c, c') \in \Lambda^+$ is well-defined where $\mathrm{Int}_{\Lambda^+}(c, c')$ with

$\partial c = \pm\partial c'$ is understood as the Λ^+-intersection number by using by the surface-framing in $F \times 0$. Then the following identities hold.

$$(t^{-1}-1)\mathrm{Int}_{\Lambda^+}(c,c') = \mathrm{Int}_{\Lambda^+}((t-1)c,c') = S^+_{\partial}(i_*^{-1}[(t-1)c],[c']),$$

$$(t^{-1}-1)(t-1)\mathrm{Int}_{\Lambda^+}(c,c') = S^+(i_*^{-1}[(t-1)c], i_*^{-1}[(t-1)c']).$$

The following lemma is used for the present argument.

Lemma 2.2. Let $C : F \times [0,1] \to S^4 \times [0,1]$ be a smooth concordance form a surface-knot $F = C(F \times 0)$ with a spin loop system $\alpha_i, \alpha'_i\,(i = 1,2,\dots,g)$ to a surface-knot $G = C(F \times 1)$ in S^4 with a spin loop system $\beta_i, \beta'_i\,(i = 1,2,\dots,g)$. Then there is a Λ^+-isomorphism φ from the non-singular Λ^+-Hermitian form $S^+_{\partial} : BA_2(F)^+ \times BH_2(\tilde{E}(F), \partial\tilde{E}(F); Z)^+ \to \Lambda^+$ to the non-singular Λ^+-Hermitian form $S^+_{\partial} : BA_2(G)^+ \times BH_2(\tilde{E}(G), \partial\tilde{E}(G); Z)^+ \to \Lambda^+$ sending the homology classes $[\alpha_i \times 0], [\alpha'_i \times 0]\,(i = 1,2,\dots,g)$ in $H_1(\partial\tilde{E}(F); Z))$ to the homology classes $[\beta_i \times 0], [\beta'_i \times 0]\,(i = 1,2,\dots,g)$ in $H_1(\partial\tilde{E}(G); Z))$, respectively.

Proof of Lemma 2.2. Let $E(C) = \mathrm{cl}(S^4 \times [0,1] \setminus N(F \times [0,1]))$ be the exterior of the concordance C. Then $(E(C); E(F), E(G))$ is a homology cobordism with $(\partial' E(C); \partial E(F), \partial E(G))$ the product cobordism for $\partial' E(C) = \mathrm{cl}(\partial E(C) \setminus (E(F) \cup E(G))$. Then $H_*(\tilde{E}(C), \tilde{E}(F); Z)$ and $H_*(\tilde{E}(C), \tilde{E}(G); Z)$ are $(t-1)$-divisible finitely generated Λ-modules. Hence

$$H_*(\tilde{E}(C), \tilde{E}(F); Z)^+ = H_*(\tilde{E}(C), \tilde{E}(G); Z)^+ = 0.$$

Then an argument of the Λ^+-homology cobordism $(\tilde{E}(C); \tilde{E}(F), \tilde{E}(G))$ similar to the standard homology cobordism argument shows the desired result. This completes the proof of Lemma 2.2.]

The proof of Theorem 1.3 is given as follows.

2.3: Proof of Theorem 1.3. *Every surface-knot F in S^4 is concordant to a trivial surface-knot G in S^4 by a concordance sending any given spin loop basis $\alpha_i, \alpha'_i\,(i = 1,2,\dots,g)$ of F to the standard spin loop basis $\beta_i, \beta'_i\,(i = 1,2,\dots,g)$ of G.* To see this, consider a trivial surface-knot $\bar{G}$ obtained from F by adding 1-handles $h_j\,(j = 1,2,\dots,m)$ (see [5]). Let $\alpha_i, \alpha'_i\,(i = 1,2,\dots,g)$, $\gamma_j, \gamma'_j\,(j = 1,2,\dots,m)$ be a spin loop basis of $\bar{G}$ with γ_j a belt loop of h_j. By [4], there is an orientation-preserving diffeomorphism f of $(S^4, \bar{G})$ sending the spin loop basis $\alpha_i, \alpha'_i\,(i = 1,2,\dots,g)$, γ_j, γ'_j $(j = 1,2,\dots,m)$ to a standard spin loop basis of $\bar{G}$. Thus, there are 2-handles on $\bar{G}$ attached along the loops $\gamma'_j\,(j = 1,2,\dots,m)$ to obtain a trivial surface-knot G by the surgery. Then G has a spin loop basis $\beta_i, \beta'_i\,(i = 1, 2, \ldots, g)$ inherited from the spin loop basis $\alpha_i, \alpha'_i\,(i = 1, 2, \ldots, g)$. (This is a similar consideration to [14, (2.5.1), (2.5.1)].) The surgery trace gives a desired concordance. Let $\Delta_i, \Delta'_i\,(i = 1, 2, \ldots, g)$ be a standard disk-basis of G with $\partial\Delta_i = \beta_i \times 1, \partial\Delta'_i = \beta'_i \times 1\,(i = 1, 2, \ldots, g)$ in $G \times 1 \subset G \times S^1 = \partial E(G)$. Let $\tilde{\Delta}_i, \tilde{\Delta}'_i\,(i = 1, 2, \ldots, g)$ be the connected lifts of $\Delta_i, \Delta'_i\,(i = 1, 2, \ldots, g)$ to $\tilde{E}(G)$ with $\partial\tilde{\Delta}_i = \beta_i \times 0, \partial\tilde{\Delta}'_i = \beta'_i \times 0\,(i = 1, 2, \ldots, g)$ in $G \times 0 \subset G \times \boldsymbol{R} = \partial\tilde{E}(G)$. By Lemma 2.2, there are relative 2-cycles $c_i, c'_i\,(i = 1, 2, \ldots, g)$ in $BH_2(\tilde{E}(F), \partial\tilde{E}(F); Z)^+$ with $\partial c_i = \alpha_i \times 0, \partial c'_i = \alpha'_i \times 0\,(i = 1, 2, \ldots, g)$ such that the homology classes $[c_i], [c'_i]\,(i = 1, 2, \ldots, g)$ are sent to the homology classes $[\tilde{\Delta}_i], [\tilde{\Delta}'_i]\,(i = 1, 2, \ldots, g)$ in $BH_2(\tilde{E}(G), \partial\tilde{E}(G); Z)^+$ by the Λ^+-isomorphism φ. Since any pair of $c_i, c'_i\,(i = 1, 2, \ldots, g)$ except for $(c_i, c'_i), (c'_i, c_i), (i = 1, 2, \ldots, g)$ is a well-defined pair, the following identities hold.

$$\mathrm{Int}_{\Lambda^+}(c_i, c_j) = \mathrm{Int}_{\Lambda^+}(\tilde{\Delta}_i, \tilde{\Delta}_j) = 0, \quad \mathrm{Int}_{\Lambda^+}(c'_i, c'_j) = \mathrm{Int}_{\Lambda^+}(\tilde{\Delta}'_i, \tilde{\Delta}'_j) = 0$$

for all i, j and

$$\mathrm{Int}_{\Lambda^+}(c_i, c'_j) = \mathrm{Int}_{\Lambda^+}(\tilde{\Delta}_i, \tilde{\Delta}'_j) = 0, \quad \mathrm{Int}_{\Lambda^+}(c'_i, c_j) = \mathrm{Int}_{\Lambda^+}(\tilde{\Delta}'_i, \tilde{\Delta}_j) = 0$$

for all i, j with $i \neq j$. There is an element $u(t) \in \Lambda$ with $u(1) = 1$ such that the products $u(t)[c_i], u(t)[c'_i]\,(i = 1, 2, \ldots, g)$ are in $BH_2(\tilde{E}(F), \partial\tilde{E}(F); Z)$ for all i. Since $u(t)$ acts on Z^{2g} as the identity $u(1) = 1$, there are compact connected oriented proper smoothly embedded surfaces $\tilde{D}_i, \tilde{D}'_i\,(i = 1, 2, \ldots, g)$ in $\tilde{E}$ with $\partial\tilde{D}_i = \alpha_i, \partial\tilde{D}'_i = \alpha'_i \times 0\,(i = 1, 2, \ldots, g)$ in $F \times 0 \subset \partial\tilde{E}$ such that

$$u(t)[c_i] = [\tilde{D}_i], \quad u(t)[c'_i] = [\tilde{D}'_i] \quad (i = 1, 2, \ldots, g)$$

in $BH_2(\tilde{E}, \partial\tilde{E}(F); Z)$. The Λ-intersection numbers

$$\mathrm{Int}_{\Lambda}(\tilde{D}_i, \tilde{D}_j) = \mathrm{Int}_{\Lambda}(\tilde{D}'_i, \tilde{D}'_j) = 0$$

for all i, j and

$$\mathrm{Int}_{\Lambda}(\tilde{D}_i, \tilde{D}'_j) = \mathrm{Int}_{\Lambda}(\tilde{D}'_i, \tilde{D}_j) = 0$$

for every i, j with $i \neq j$. Then the proper surfaces $\tilde{D}_i, \tilde{D}'_i\,(i = 1, 2, \ldots, g)$ in $\tilde{E}$ are modified without changing the boundary loops into higher genus surfaces which are embeddable into E under the covering projection $\mathrm{proj} : \tilde{E} \to E$ by [10, Theorem 4.1]. By writing $\mathrm{proj}(\tilde{D}_i), \mathrm{proj}(\tilde{D}'_i)\,(i = 1, 2, \ldots, g)$ as $D_i, D'_i\,(i = 1, 2, \ldots, g)$, a surface-basis $D_i, D'_i\,(i = 1, 2, \ldots, g)$ of F in S^4 is obtained. This completes the proof of Theorem 1.3.

3. The infinite cyclic covering homology of the surface-knot manifold

Let $D_i, D'_i\,(i=1,2,\dots,g)$ be a surface-basis of F in E with $\partial D_i = \alpha_i \times 1, \partial D'_i = \alpha'_i \times 1\,(i=1,2,\dots,g)$ in $F \times 1 \subset F \times S^1 = \partial E$ by Theorem 1.3. Let V_0 be a handlebody of genus g with $\partial V_0 = F$ such that $\alpha_i\,(i=1,2,\dots,g)$ bound disjoint disks in V_0. The *surface-knot manifold* of a surface-knot F in S^4 is the 4D manifold $M = E \cup V_0 \times S^1$ obtained from S^4 by replacing $N(F) = F \times D^2$ with $V_0 \times S^1$. Then $H_1(M;Z) \cong Z$. Let $a(D'_i) = T_i \cup D'_i\,(i=1,2,\dots,g)$ be the 2-cycles in M homologous to $T_i\,(i=1,2,\dots,g)$, and $s(D_i) = D_i \cup d_i \times 1\,(i=1,2,\dots,g)$ the closed connected oriented surfaces for disjoint disks d_i in $V_0 \times 1$ with $\partial d_i = \alpha_i\,(i=1,2,\dots,g)$. The second homology $H_2(M;Z)$ is a free abelian group of rank $2g$ with a basis consisting of the homology classes $[a(D'_i)], [s(D_i)]\,(i=1,2,\dots,g)$ with intersection numbers

$$\mathrm{Int}([a(D'_i)],[a(D'_j)]) = \mathrm{Int}([s(D_i)],[s(D_j)]) = 0,$$

$$\mathrm{Int}([a(D_i)],[s(D_j)]) = \mathrm{Int}([s(D_i)],[a(D_j)]) = \delta_{ij}$$

for all i,j.

Let $\mathrm{proj}: \tilde{M} \to M$ be the infinite cyclic covering of M with $\tilde{M} = \tilde{E} \cup V_0 \times \boldsymbol{R}$. Let $\tilde{D}_i, \tilde{D}'_i\,(i=1,2,\dots,g)$ be the connected lifts of $D_i, D'_i\,(i=1,2,\dots,g)$ with $\partial \tilde{D}_i = \alpha_i \times 0, \partial \tilde{D}'_i = \alpha'_i \times 0\,(i=1,2,\dots,g)$ in $F \times 0 \subset F \times \boldsymbol{R} = \partial \tilde{E}$. Let

$$a(\tilde{D}'_i) = (-\tilde{D}'_i) \cup [0,1] \cup t\tilde{D}'_i, \quad s(\tilde{D}_i) = \tilde{D}_i \cup d_i \times 0 \qquad (i=1,2,\dots,g)$$

be the closed connected oriented surfaces in $\tilde{M}$. Let

$$x'_i = [a(\tilde{D}'_i)], \quad y_i = [s(\tilde{D}_i)] \qquad (i=1,2,\dots,g)$$

be the homology classes in $H_2(\tilde{M};Z)$. Let X be the Λ-submodule of $BH_2(\tilde{M};Z)$ generated over Λ by the elements $x \in BH_2(\tilde{M};Z)$ with $S^B(x, x'_i) = 0$ for all i, and Y the Λ-submodule of $BH_2(\tilde{M};Z)$ generated over Λ by the elements $y_i\,(i=1,2,\dots,g)$. The following lemma is shown.

Lemma 3.1. There is a direct sum splitting $E^0E^0(X) \oplus Y$ of the free Λ-module $E^0E^0(BH_2(\tilde{M};Z))$ with $y_i\,(i=1,2,\dots,g)$ a Λ-basis of Y such that the Λ-Hermitian form

$$S: E^0E^0(BH_2(\tilde{M};Z)) \times E^0E^0(BH_2(\tilde{M};Z)) \to \Lambda$$

is given by

$$S(x_i,x_j) = S(y_i,y_j) = 0, \quad S(x_i,y_j) = \delta_{ij} \quad (i,j=1,2,\dots,g)$$

for a Λ-basis $x_i\,(i=1,2,\dots,g)$ of $E^0E^0(X)$.

Proof of Lemma 3.1. By construction, $S(x'_i, x'_j) = S(y_i, y_j) = S(x'_i, y_j) = 0\,(i \neq j)$ and $S(x'_i, y_i) = 1 + (t-1)f_i(t)$ for some $f_i(t) \in \Lambda$. Let X_i be the quotient rank one Λ-module of X by the maximal submodule generated over Λ by x'_j for all $j \neq i$, so that X_i is a torsion-free Λ-module of rank one and $E^0(X_i) \cong \Lambda$. Let $q_i \in E^0(X_i)$ be the Λ-homomorphism sending $x \in X_i$ to $S(x, y_i) \in \Lambda$. Then it is shown that the element q_i is a generator of $E^0(X_i) \cong \Lambda$. To see this, under the identification $E^0(X_i) = \Lambda$, suppose q_i is a non-unit element $q_i = q_i(t)$ in Λ. Then $q_i(1) = \pm 1$ since $q_i(t)$ divides the polynomial $1 + (t-1)f_i(t)$. Let p be a prime number such that $q_i(t)$ is still a non-unit polynomial in the principal ideal domain $\Lambda_p = Z_p[t, t^-1]$ and the first Z-torsion product $\mathrm{Tor}_1(H_1(\tilde{M}; Z), Z_p) = 0$ by using that the Z-torsion Λ-submodule of $H_1(\tilde{M}; Z)$ is finite because $H_1(\tilde{M}; Z)$ is $(t-1)$-indivisible. Then the universal coefficient theorem means $H_2(\tilde{M}; Z_p) = H_2(\tilde{M}; Z) \otimes Z_p$. Hence $X \otimes Z_p$ is a self-orthogonal complement with respect to the nonsingular Λ_p-intersection form

$$S_p : BH_2(\tilde{M}; Z_p) \times BH_2(\tilde{M}; Z_p) \to \Lambda_p$$

in [6]. This means that there is an element x''_i in X_i such that $S(x''_i, y_i) = 1 + pg_i(t)$ for some element $g_i(t) \in \Lambda$, so that $q_i(t)$ must be a unit element in Λ_p, which contradicts that $q_i(t)$ is a non-unit element in Λ_p. Thus, q_i is a unit element in Λ. Let $\bar{q}_i \in E^0(X)$ be the image of q_i under the natural monomorphism $E^0(X_i) \to E^0(X)$. Then the elements $\bar{q}_i\,(i = 1, 2, \ldots, g)$ form a Λ-basis for $E^0(X)$. In fact, for every element $q \in E^0(X)$, let $q(x'_i) = c_i(t)$ be the element of Λ. Then $q = \sum_{i=1}^{g} c_i(t)\bar{q}_i$. If $\sum_{i=1}^{g} c'_i(t)\bar{q}_i = 0$, then $c'_i(t)\bar{q}_i(x'_i) = c'_i(t)(1 + (t-1)f_i(t)) = 0$ and $c'_i = 0$ for all i. Let $\bar{q}^*_i \in E^0E^0(X)\,(i = 1, 2, \ldots, g)$ be the dual basis of $\bar{q}_i\,(i = 1, 2, \ldots, g)$of $E^0(X)$. Since $S(\bar{q}^*_i, \bar{q}_j) = S(y_i, y_j) = 0$ and $S(\bar{q}^*_i, y_j) = \delta_{ij}$ for all i, j, the elements $x_i = \bar{q}^*_i, y_i\,(i = 1, 2, \ldots, g)$ form a desired Λ-basis for $E^0E^0(BH_2(\tilde{M}; Z)) = E^0E^0(X) \oplus Y$. This completes the proof of Theorem 3.1.

The following corollary is obtained from the proof of Lemma 3.1.

Corollary 3.2. For the elements $x'_i = [a(\tilde{D}'_i)], y_i = [s(\tilde{D}_i)]\,(i = 1, 2, \ldots, g)$ in $X \oplus Y = BH_2(\tilde{M}; Z)$, an element $x \in BH_2(\tilde{M}; Z)$ belongs to the direct summand X if and only if the product $u(t)x$ for an element $u(t) \in \Lambda$ with $u(1) = \pm 1$ is a linear combination of $x'_i\,(i = 1, 2, \ldots, g)$ with coefficients in Λ.

Proof of Corollary 3.2. In the proof of Lemma 2.2, the identities $(1 + (t-1)f_i(t))x_i = x'_i\,(i = 1, 2, \ldots, g)$ hold, so that if $x \in BH_2(\tilde{M}; Z)$ is in X, then the product $u(t)x$ for some $u(t)$ with $u(1) = \pm 1$ is a linear combination of $x'_i\,(i = 1, 2, \ldots, g)$ with coefficients in Λ. Conversely, since X is self-orthogonal with respect to the non-degenerate Λ-intersection form $S_M : BH_2(\tilde{M}; Z) \times BH_2(\tilde{M}; Z) \to \Lambda$ and every linear combination of x'_i $(i = 1, 2, \ldots, g)$ with coefficients in Λ is in X, if $u(t)x$ for some $u(t) \in \Lambda$ with $u(1) = \pm 1$ is in X, then x is in X. This completes the proof of Corollary 3.2.

4. Proofs of Theorems 1.1 and Corollary 1.2

The following lemma is a classification of finitely generated torsion-free Λ-modules.

Lemma 4.1. For a finite Λ-module D, let $[\omega_i]$ $(i = 1, 2, \ldots, n_r)$ be all the r-classes on D, and B_i a finitely generated torsion-free Λ-module of rank r given by the kernel $\ker(\omega_i : \Lambda^r \to D)$ for every i. Then B_i $(i = 1, 2, \ldots, n_r)$ are mutually distinct up to Λ-isomorphisms and every finitely generated torsion-free Λ-module B of rank r with $E^2E^1(B) \cong D$ is Λ-isomorphic to B_i for some i. Further, any two finitely generated torsion-free Λ-modules B and B' with $E^2E^1(B) \cong E^2E^1(B')$ are stably Λ-isomorphic, i.e., $B \oplus \Lambda^m \cong B' \oplus \Lambda^{m'}$ for some non-negative integers m, m'.

Proof of Lemma 4.1. For a finitely generated torsion-free Λ-module B with $E^2E^1(B) \cong D$, there is a short exact sequence $0 \to B \to E^0E^0(B) \to E^2E^1(B) \to 0$. Since there are Λ-isomorphisms $g_B : E^0E^0(B) \to \Lambda^r$ and $g_D : E^2E^1(B) \to D$ to define an r-weight $\omega_B : \Lambda^r \to D$ whose kernel $\mathrm{Ker}(\omega_B) = B_D$ is Λ-isomorphic to B, If there is a Λ-isomorphism $f : B \to B'$, then the Λ-isomorphism f induces a Λ-isomorphism from the short exact sequence $0 \to B \to E^0E^0(B) \to E^2E^1(B) \to 0$ to the short exact sequence $0 \to B' \to E^0E^0(B') \to E^2E^1(B') \to 0$. Hence there are equivalent r-weights $\omega_B, \omega_{B'} : \Lambda^r \to D$ with kernels $\mathrm{Ker}(\omega_B) = B_D \cong B$, $\mathrm{Ker}(\omega_{B'}) = B'_D \cong B'$. For $\mathrm{Ker}(\omega) = B_D$, the inclusion $B_D \subset \Lambda^r$ induces a Λ-isomorphism $g_B : E^0E^0(B_D) \cong \Lambda^r$. Hence there is a Λ-isomorphism $g_D : E^2E^1(B_D) \to D$ to define an r-weight $\omega_{B_D} : \Lambda^r \to D$ which is equivalent to ω. For equivalent r-weights $\omega, \omega' : \Lambda^r \to D$ with $\mathrm{Ker}(\omega) = B_D$ and $\mathrm{Ker}(\omega') = B'_D$, the five lemma for a short exact sequence shows that B_D is Λ-isomorphic to B'_D. From finiteness of the Λ-module $\hom_\Lambda(\Lambda^r, D)$, a desired system of finitely generated torsion-free Λ-module B_i $(i = 1, 2, \ldots, n_r)$ of rank r with $E^2E^1(B_i) \cong D$ is obtained. Let $\boldsymbol{e} = \{e_i | \, i = 1, 2, \ldots, r\}$ be a standard Λ-basis of Λ^r. For an r-weight $\omega : \Lambda^r \to D$, assume that $\omega(e_1) = \omega(e_i)$ for some $i \neq 1$. Then replace the basis element e_i with $e_i - e_1$. By continuing this process, there is a Λ-isomorphism $f_\Lambda : \Lambda^r \to \Lambda^r$ such that $\omega' = f_\Lambda \omega$ is an r-weight such that ω' sends a Λ-subbase $\boldsymbol{e}'$ of $\boldsymbol{e}$ injectively and the remaining Λ-subbasis $\boldsymbol{e} \setminus \boldsymbol{e}'$ to 0. This means that $\mathrm{Ker}(\omega) = B_D$ is Λ-isomorphic to $B' \oplus \Lambda^{r-r'}$ for $B' = \Lambda^{r'} \cap \mathrm{Ker}(\omega')$ for the Λ-submodule $\Lambda^{r'}$ given by the Λ-subbasis $\boldsymbol{e}'$. For an r-weight $\omega : \Lambda^r \to D$, assume that ω sends $\boldsymbol{e}$ injectively to D. Let $n = |D|$. Let $\bar{\omega} : \Lambda^n \to D$ be a Λ-epimorphism extending ω so that the standard basis $\bar{\boldsymbol{e}}$ of Λ^n bijectively to D. For every basis element e_j in $\bar{\boldsymbol{e}} \setminus \boldsymbol{e}$, write

$$\bar{x}_i(e_j) = \sum_{k=1}^{r} a_{jk}(t)\xi(e_k) \quad (a_{jk}(t) \in \Lambda).$$

Under the new basis of Λ^n obtained by replacing every e_j with $e_j - \sum_{k=1}^{r} a_{jk}(t)e_k$, the kernel of $\bar{\omega}$: is Λ-isomorphic to $B_D \oplus \Lambda^{n-r}$. Note that any n-weight $\bar{\omega} : \Lambda^n \to D$ sending the standard basis $\bar{\boldsymbol{e}}$ of Λ^n bijectively to D gives the unique class $[\bar{\omega}]$. Thus, any s-weight $\omega' : \Lambda^s \to D$ sending the standard basis $\boldsymbol{e}'$ of Λ^n injectively to D, so that there is a Λ-isomorphism $B_D \oplus \Lambda^{n-r} \cong B'_D \oplus \Lambda^{n-r'}$ for $B'_D = \mathrm{Ker}(\omega')$. This completes the proof of Lemma 4.1.

4.2: Proof of Theorem 1.1. For the proof of (1), let $H = R_1(F)$ for a surface-knot F of genus g. By the second duality of [7], $E^2(R_1(F)) = E^2(DA_1(F)/\Theta(F)) \cong E^2E^1(BA_2(F)$. Hence there is a t-anti Λ-isomorphism

$$E^2(H) = E^2(R_1(F)) \cong E^2E^1(BA_2(F)) = E^2E^1(X \oplus Y)$$

by assuming (3). Since $E^0E^0(X)$ and Y are free Λ-modules of rank g and there is a Λ-epimorphism $E^0E^0(X) \to E^2E^1(X)$, the following inequalities hold.

$$e(E^2E^1(X \oplus Y)) = e(E^2E^1(X)) \leq e(E^0E^0(X)) = g.$$

Thus, $e(H) \leq g$, assuming (3). Conversely, let H be a $(t-1)$-divisible finitely generated Λ-module with inequality $e(E^2(H)) \leq g$. Then H is the first module $A_1(F)$ of a ribbon surface-knot F of genus g in S^4 with $\Theta(F) = 0$ by [13]. Thus, $H = A_1(F) = R_1(F)$, which shows (1) by assuming (3). For the proof of (2), let $[\omega_F]$ be the g-class on the finite Λ-module $E^2E^1(BA_2(F))$, which is t-anti Λ-isomorphic to $E^2(R_1(F))$, so that $[\omega_F]$ is considered as a g-class on the finite Λ-module $E^2(R_1(F))$. By Lemmas 4.1, $BA_2(F)$ is determined by this g-class on $E^2(R_1(F))$. By the first duality of [7], the torsion Λ-module $TA_2(F) = T_DA_2(F)$ is t-anti Λ-isomorphic to $E^1(T_DH_1(\tilde{E}, \partial\tilde{E}; Z)) = E^1(T_DA_1(F)) = E^1(T_DR_1(F)) = E^1(R_1(F))$ by Lemma 2.1, showing (2). For the proof of (3), note that the zeroth duality of [7] means that there is a *non-singular* Λ-form

$$S_\partial : E^0E^0(BA_2(F)) \times E^0E^0BH_2(\tilde{E}, \partial\tilde{E}; Z) \to \Lambda$$

extending the non-degenerate Λ-intersection form

$$S_\partial^B : BA_2(F) \times BH_2(\tilde{E}, \partial\tilde{E}; Z) \to \Lambda$$

which also defines a *non-degenerate* Λ-Hermitian Λ-intersection form $S^B : BA_2(F) \times BA_2(F) \to \Lambda$. Since $DH_2(\tilde{M}; Z) = 0$ and $TH_2(\tilde{M}; Z) \cong E^1(H_1(\tilde{M}; Z))$ is $(t -$

1)-divisible by a method similar to the proof of Lemma 2.1, and $H_2(\tilde{M},\tilde{E};Z) = H_2((V_0,F)\times \boldsymbol{R};Z)\cong Z^g$, there is a natural exact sequence

$$0\to BA_2(F)\overset{i_*}{\to} BH_2(\tilde{M};Z)\overset{j_*}{\to} Z^g\to 0,$$

which induces a natural exact sequence

$$0\to E^0E^0(BA_2(F))\to E^0E^0(BH_2(\tilde{E},\partial\tilde{E};Z))\to Z^g\to 0.$$

By Lemma 3.1 and Corollary 3.2, $BH_2(\tilde{M};Z) = X\oplus Y$, where Y is a free Λ-module and X is characterized by the Λ-submodule of $BH_2(\tilde{M};Z)$ consisting of an element x such that the product $u(t)x$ for an element $u(t)\in\Lambda$ with $u(1)=1$ is a linear combination of $x'_i\,(i=1,2,\dots,g)$. If $x\in X$ has $j_*(u(t)x)=0$, then

$$j_*(u(t)x) = u(t)j_*(x) = u(1)j_*(x) = j_*(x) = 0$$

and $x\in \mathrm{Ker}(i_*) = BA_2(F)$. Let $X_F = i_*^{-1}(X)$, which is characterized by the Λ-submodule of $BA_2(F)$ consisting of an element x such that the product $u(t)x$ for an element $u(t)\in\Lambda$ with $u(1)=1$ is a linear combination of $x'_i = [a(\tilde{D}'_i)]\,(i=1,2,\dots,g)$ regarded as elements of $BA_2(F)$. This means that i_* defines a Λ-isomorphism $X_F\cong X$. Let $Y_F = i_*^{-1}(Y)$ which is a free Λ-module with basis $[a(\tilde{D}_i)]$, $(i=1,2,\dots,g)$ since $i_*([a(\tilde{D}_i)]) = (t-1)[s(\tilde{D}_i]$ in $H_2(\tilde{M};Z)$. This means that i_* defines a natural excat sequence

$$0\to Y_F\to Y\to Z^g\to 0.$$

Then $BA_2(F) = X_F\oplus Y_F$ and the non-degenerate Λ-Hermitian form

$$S: E^0E^0(BA_2(F))\times E^0E^0(BA_2(F))\to\Lambda$$

is given by

$$S(x_i,x_j) = S(y_i,y_j) = 0,\quad S(x_i,y_j) = (t-1)\delta_{ij}\quad (i,j=1,2,\dots,g)$$

for a Λ-basis $x_i,y_i\,(i=1,2,\dots,g)$ of $E^0E^0(BA_2(F)) = E^0E^0(X_F)\oplus Y_F$ with $x_i\,(i=1,2,\dots,g)$ a Λ-basis of $E^0E^0(X_F)$ and $y_i\,(i=1,2,\dots,g)$ a Λ-basis of Y_F, showing (3). To see (4), note that $A_3(F) = TA_3(F)$ since $H_3(E;Z)=0$ means that $A_3(F)$ is $(t-1)$-divisible. By the first duality of [7], $T_DA_3(F)$ is t-anti Λ-isomorphic to $\hom_\Lambda(T_DH_0(\tilde{E},\partial\tilde{E};Z),Q(\Lambda)/\Lambda)$ which is 0. By the second duality of [7], $DA_3(F)$ is t-anti Λ-isomorphic to $E^1(BH_0(\tilde{E},\partial\tilde{E};Z))$ which is 0. Thus, $A_3(F)=0$, showing (4). This completes the proof of Theorem 1.1.

In the similar way to the proof of (4) in 4.2, it is shown that $H_3(\tilde{E},\partial\tilde{E};Z)\cong Z$ whose integral generator is the fundamental class of the infinite cyclic connected covering $\tilde{E} \to E$ represented by a leaf of the surface-knot F (see [9]). In fact, by the first duality of [7], $H_3(\tilde{E}, \partial\tilde{E}; Z) = T_D H_3(\tilde{E}, \partial\tilde{E}; Z)$ which is t-anti Λ-isomorphic to $E^1(A_0(F)) \cong Z$. The proof of Corollary 1.2 is done as follows.

4.3: Proof of Corollary 1.2. Since π is the group of ribbon presentation, of deficiency 0, there is a ribbon torus-knot T in S^4 with $\pi_1(S^4 \backslash T, x_0) = \pi$ and $A_1(T) = D$ (see [12]). Since D is a $(t-1)$-divisible finite Λ-module with $e(D) = 1$, the first module $A_1(T_g)$ of T_g in S^4 is the finite Λ-module D^g, the direct sum of g copies of D, and $E^2(D^g)$ is seen to be Λ-isomorphic to D^g and $e(D^g) = g$ since p is a prime number. For $p \geq 5$, the finite Λ-module D^g does not admit any t-anti Λ-automorphism, so that $\Theta(F) = 0$ and $A_1(F) = R_1(F)$ for any surface-knot F in S^4 with $A_1(F) = D^g$. Since $e(R_1(F)) = g$, the reduced first module $R_1(F)$ is not Λ-isomorphic to the reduced first module of any surface-knot of genus $g' < g$ by Theorem 1.1 (1), so that π is not the fundamental group of any surface-knot of genus $g' < g$. This completes the proof of Corollary 1.2.

5. An exact leaf and the torsion-linking of a surface-knot

Let V'_F be a leaf of a surface-knot F in S^4 containing a half surface-basis $D'_i\,(i = 1, 2, \ldots, g)$ of a surface-basis $D_i, D'_i\,(i = 1, 2, \ldots, g)$ as proper surfaces. Let W be a compact connected oriented 3-manifold with $\partial W = F$, and $V^* = V'_F \cup W$ be the closed oriented 3-manifold obtained from V_F and W by pasting along F with an orientation-reversing diffeomorphism of F. The following lemma is used for the proof of Theorem 1.4.

Lemma 5.1. If $H_1(W; Z)$ is a free abelian group and the loop system $\alpha_i\,(i = 1, \ldots, g)$ or $\alpha'_i\,(i = 1, \ldots, g)$ in F represents a basis of the image of the boundary homomorphism $\partial_* : H_2(W, F; Z) \to H_1(F; Z)$, then the inclusion $V'_F \to V^*$ induces an isomorphism $\mathrm{Tor}H_1(V'_F; Z) \to \mathrm{Tor}H_1(V^*; Z)$.

Proof of Lemmay 5.1. Since the exact leaf V'_F contains the disjoint proper surfaces $C'_i\,(i = 1, 2, \ldots, g)$, there is a retraction $r_F : V'_F \to \gamma$ for a legged loop system γ with the loops $\alpha_i\,(i = 1, 2, \ldots, g)$ in F such that the composite $r_F i_F : \gamma \to \gamma$ for the inclusion $i_F : \gamma \to V'_F$ is homotopic to the identity. Then the homology exact sequence

$$H_2(V'_F, F; Z) \to H_1(\partial F; Z) \to H_1(V'_F; Z) \to H_1(V'_F, F; Z) \to 0$$

induces a split short exact sequence

$$0 \to Z^g \to H_1(V'_F; Z) \to H_1(V'_F, F; Z) \to 0,$$

where Z^g denotes a free abelian group with basis represented by the loops $\alpha_i\,(i = 1, 2, \ldots, g)$. Hence there are natural isomorphisms

$$\mathrm{Tor}H_1(V'_F; Z) \cong \mathrm{Tor}(H_1(V'_F; Z)/\mathrm{Im}i_*) \cong\cong \mathrm{Tor}H_1(V^*, W; Z)$$

for the image $\mathrm{Im}i_* = \mathrm{Im}(i_* : H_1(F; Z) \to H_1(V'_F; Z))$. Since

$$H_1(V^*, V'_F; Z) \cong H_1(W, F; Z) \cong H^2(W; Z)$$

is a free abelian group and the image $\mathrm{Im}\partial'_* = \mathrm{Im}(\partial'_* : H_2(V^*, V'_F; Z) \to H_1(V'_F; Z))$ is equal to the image $\mathrm{Im}i_*\partial_* = \mathrm{Im}(i_*\partial_* : H_2(W, F; Z) \to H_1(F; Z) \to H_1(V'_F; Z))$, the exact sequence

$$H_2(V^*, V'_F; Z) \to H_1(V'_F; Z) \to H_1(V^*; Z) \to H_1(V^*, V'_F; Z) \to 0$$

induces a natural isomorphism

$$\mathrm{Tor}(H_1(V'_F; Z)/\mathrm{Im}\partial'_*) \cong \mathrm{Tor}H_1(V^*; Z).$$

If the loop system $\alpha_i\,(i = 1, 2, \ldots, g)$ or $\alpha'_i\,(i = 1, 2, \ldots, g)$ represents a basis of $\mathrm{Im}\partial_*$ in $H_1(F; Z)$, then there is a natural isomorphism

$$\mathrm{Tor}(H_1(V'_F; Z)/\mathrm{Im}\partial_*) \to \mathrm{Tor}(H_1(V'_F; Z)/\mathrm{Im}i_*).$$

Hence the inclusion $V'_F \to V^*$ induces an isomorphism $\mathrm{Tor}H_1(V'_F; Z) \to \mathrm{Tor}H_1(V^*; Z)$. This completes the proof of Lemma 5.1.

Theorem 1.4 is shown as follows.

5.2: Proof of Theorem 1.4. Let V_F be any leaf of E with $\partial V_F = F \times 1$ in $F \times S^1 = \partial E$, and $D_i, D'_i\,(i = 1, 2, \ldots, g)$ any surface-basis of F in E with $\partial D_i = \alpha_i \times 1, \partial D'_i = \alpha'_i \times 1$. Let $G_k\,(k = 1, 2, \ldots, s)$ be closed connected oriented surfaces in E representing Λ-generators of the direct summand X_F of $BA_2(F)$ in Theorem 1.1 (2), which can be disjointedly embedded in E under the covering projection $\mathrm{proj} : \tilde{E} \to E$ by [10, Theorem 4.1] because $S(X_F, X_F) = 0$. Since V_F, D_i, G_k are all trivially liftable in $\tilde{E}$, the leaf V_F is modified so that the interior $\mathrm{Int}D_i$ of D_i transversely meets V_F in disjoint simple loops each of which is null-homologous in D_i and G_i transversely meets V_F in disjoint simple loops each of which is null-homologous in G_k. Let D^0_i be an innermost piece of the surfaces of D_i divided by the loops $V_F \cap D_i$. Take a normal disk bundle $D^0_i \times D^2$ of D^0_i in E with $(\partial D^0_i) \times D^2$ a normal disk bundle of the loop ∂D^0_i in V_F and replace $(\partial D^0_i) \times D^2$ with $D^0_i \times S^1$ to obtain from V_F to obtain a new leaf of F in E. By continuing this process, the leaf V_F is modified to have

$V_F \cap \mathrm{Int}D_i = \emptyset\,(i = 1, 2, \ldots, g)$. By continuing the same process after pushing the $\alpha_i \times 1$ into the interior of V_F, the 3-manifolds $D_i \times S^1\,(i = 1, 2, \ldots, g)$ are contained in the resulting leaf V_F of E. By the similar modification, V_F is modified so that a normal circle bundle $G_k \times S^1$ of G_k is made disjoint from V_F. Replace V_F with a connected sum of V_F and $G_k \times S^1\,(k = 1, 2, \ldots, s)$ in E. To show that the resulting leaf V_F is a desired leaf of F in S^4, let $V = V_F \cup V_0 \times 1$ be a closed leaf in the surface-knot manifold $M = E \cup V_0 \times S^1$, where V_0 is a handlebody with a disjoint disk system $d_i\,(i = 1, 2, \ldots, g)$ bounded by the half loop basis $\alpha_i\,(i = 1, 2, \ldots, g)$. Then the surface D_i extends to a closed surface $s(D_i) = D_i \cup d_i \times 1$ in V. By 4.2, $BA_2(F) = X_F \oplus Y_F$, $BH_2(\tilde{M}) = X \oplus Y$ and the short exact sequence

$$0 \to E^0E^0(BA_2(F)) \to E^0E^0(BH_2(\tilde{M})) \to Z^g \to 0$$

splits into the isomorphism $X_F \cong X$ and the short exact sequence $0 \to Y_F \to Y \to Z^{2g} \to 0$. Hence the natural homomorphism $H_2(\tilde{V}_F; Z) \to E^0E^0(BH_2(\tilde{E}))$ with image X_F induces the natural homomorphism $H_2(\tilde{V}; Z) \to E^0E^0(BH_2(\tilde{M}))$ with image X. By [10], the closed leaf V of M is a closed exact leaf of M, meaning that the following natural sequence

$$(*) \qquad 0 \to \mathrm{Tor}H_2(\tilde{M}, \tilde{V}; Z) \to \mathrm{Tor}H_1(\tilde{V}; Z) \to \mathrm{Tor}H_1(\tilde{M}; Z)$$

is an exact sequence on integral torsions. By Lemma 5.1, there is a natural isomorphism $\mathrm{Tor}H_1(\tilde{V}_F; Z) \cong \mathrm{Tor}H_1(V; Z)$. Since $H_1(\tilde{E}; Z)$ and $H_1(\tilde{M}; Z)$ are $(t-1)$-divisible and $(t-1)H_k(\tilde{M}, \tilde{E}; Z) = 0\,(k = 1, 2)$, there is a natural Λ-isomorphism $H_1(\tilde{E}; Z) \to H_1(\tilde{M}; Z)$. Further, there is a natural Λ-isomorphism $\mathrm{Tor}H_2(\tilde{E}, \tilde{V}_F; Z) \to \mathrm{Tor}H_2(\tilde{M}, \tilde{V}; Z)$. In fact, there are Λ-isomorphisms $H_2(\tilde{E}, \tilde{V}_F; Z) \cong H_2(\tilde{E} \cup \tilde{V}, \tilde{V}; Z)$ and $H_k(\tilde{M}, \tilde{E} \cup \tilde{V}; Z) \cong H_k(\tilde{V}_0 \times (I, \partial I); Z) \cong H_{k-1}(\tilde{V}_0; Z)$ by the excision theorem, where I denotes the interval $[0, 1]$. Since $H_2(\tilde{V}_0; Z) = 0$ and $H_1(\tilde{V}_0; Z) \cong \Lambda^g$, there is a natural exact sequence

$$0 \to H_2(\tilde{E} \cup \tilde{V}, \tilde{V}; Z) \to H_2(\tilde{M}, \tilde{V}; Z) \to \Lambda^g,$$

which implies a natural Λ-isomorphism $\mathrm{Tor}H_2(\tilde{E}, \tilde{V}_F; Z) \cong \mathrm{Tor}H_2(\tilde{M}, \tilde{V}; Z)$ as desired. Thus, the natural sequence

$$0 \to \mathrm{Tor}H_2(\tilde{E}, \tilde{V}_F; Z) \to \mathrm{Tor}H_1(\tilde{V}_F; Z) \to \mathrm{Tor}H_1(\tilde{E}; Z)$$

is equivalent to the exact sequence $(*)$ and V_F is an exact leaf of E. This completes the proof of Theorem 1.4.

The proof of Corollary 1.5 is given as follows.

5.3: Proof of Corollary 1.5. Since V is a closed exact leaf of M, it is shown in [11] that the linking $\ell_V : \mathrm{Tor}H_1(V;Z) \times \mathrm{Tor}H_1(V;Z) \to Q/Z$ is isomorphic to the orthogonal sum of the torsion linking $\ell_M : D^\theta H_1(\tilde{M};Z) \times D^\theta H_1(\tilde{M};Z) \to Q/Z$ given by the second duality of [7] and a hyperbolic linking. Because $DA_1(F) \cong DH_1(\tilde{M};Z)$ and $E^1(BA_2(F)) \cong E^1(BH_2(\tilde{M};Z))$ as $(t-1)$-divisible finite modules, the torsion linking ℓ_M is Λ-isomorphic to the torsion linking $\ell_F : \Theta(F) \times \Theta(F) \to Q/Z$ by the second duality of [7]. By Lemma 5.1, the linking $\ell_{V_F} : \mathrm{Tor}H_1(V_F;Z) \times \mathrm{Tor}H_1(V_F;Z) \to Q/Z$ is non-singular and isomorphic to ℓ_V. Thus, the linking ℓ_{V_F} is an orthogonal sum of ℓ_F and a hyperbolic linking. This completes the proof of Corollary 1.5.

Acknowledgements This work was partly supported by JSPS KAKENHI Grant Number JP21H00978 and MEXT Promotion of Distinctive Joint Research Center Program JPMXP0723833165.

References

[1] M. Farber, Duality in an infinite cyclic covering and even-dimensional knots, Math. USSR-Izv. 11 (1977), 749-781.

[2] H. Gluck, The embedding of 2-spheres in the four-sphere, Trans. Amer. Math. Soc. 104 (1962), 308-333.

[3] J. A. Hillman and A. Kawauchi, Unknotting orientable surfaces in the 4-sphere, J. Knot Theory Ramifications 4(1995), 213-224.

[4] S. Hirose, On diffeomorphisms over surfaces trivially embedded in the 4-sphere, Algebraic and Geometric Topology 2 (2002), 791-824.

[5] F. Hosokawa and A. Kawauchi, Proposals for unknotted surfaces in four-space, Osaka J. Math. 16 (1979), 233-248.

[6] A. Kawauchi, On quadratic forms of 3-manifolds, Invent. Math. 43 (1977), 177-198.

[7] A. Kawauchi, Three dualities on the integral homology of infinite cyclic coverings of manifolds, Osaka J. Math. 23(1986), 633-651.

[8] A. Kawauchi, A survey of knot theory, Birkhäuser (1996).

[9] A. Kawauchi, On the fundamental class of an infinite cyclic covering, Kobe J. Math. 15 (1998), 103-114.

[10] A. Kawauchi, Algebraic characterization of an exact 4-manifold with infinite cyclic first homology, Journal Atti Sem. Mat. Fis. Univ. Modena 48 (2000), 405-424.

[11] A. Kawauchi, Torsion linking forms on surface-knots and exact 4-manifolds, in: Knots in Hellas '98, Series on Knots and Everything 24 (2000), 208-228, World Sci. Publ.

[12] A. Kawauchi, On the surface-link groups, Intelligence of low dimensional topology 2006, Series on knots and everything 40 (2007), 157-164, World Sci. publ.

[13] A. Kawauchi, The first Alexander Z[Z]-modules of surface-links and of virtual links, Geometry & Topology Monographs 14 (2008), 353-371.

[14] A. Kawauchi, Ribbonness of a stable-ribbon surface-link, I. A stably trivial surface-link, Topology and its Applications 301(2021), 107522 (16pages).

[15] A. Kawauchi and S. Kojima, Algebraic classification of linking pairings on 3-manifolds, Math. Ann. 253 (1980), 29-42.

[16] A. Kawauchi, T. Shibuya and S. Suzuki, Descriptions on surfaces in four-space, I : Normal forms, Math. Sem. Notes, Kobe Univ. 10(1982), 75-125; II: Singularities and cross-sectional links, Math. Sem. Notes, Kobe Univ. 11(1983), 31-69.

[17] M. A. Kervaire, Les noeuds de dimensions suprérieures, Bull. Soc. Math. France 93 (1965), 225-271.

[18] J. Levine, Knot modules. I, Trans. Amer. Math. Soc. 229 (1977), 1-50.