

# TGTC Seminar

日時:2023年7月14日(金) 17:00-18:30

場所:大阪公立大学杉本キャンパス 理学部F棟 小講究室B(F405)

講演者:Anton Ayzenberg (Higher School of Economics, Moscow)

講演題目:The poset of all topologies on a finite set and nerves beyond nerve theorem  
(work in progress joint with M. Beketov and G. Magai)

講演要旨: Motivated by some problems in automated machine learning, we investigated two objects: the space of all directed acyclic graphs on  $n$  vertices and the space of non strongly connected directed graphs on  $n$  vertices. First space appears to be homotopy equivalent to a sphere of dimension  $n-2$ , while the second is the wedge of  $(n-1)!$  many  $(2n-4)$ -dimensional spheres. The second space is also homotopy equivalent to the lattice of all topologies on  $n$  points. Both results date back to Bjorner and Welker, but their poset-theoretical manifestation also appears in the works of Bouc.

There are two classes of simplicial complexes which generalize these two wonderful spaces. First class constitutes the nerves of coverings of a sphere by open hemispheres. Such complexes may be identified with combinatorial Alexander duals of convex polytopes via Gale duality. Their homotopy type is easily described by the Nerve theorem. The second class is given by the nerves of the coverings of a sphere by closed hemispheres. In this case, the Nerve theorem is not applicable, but we described the homotopy type of nerves anyway.

It is interesting that our result about closed hemispheres explains some mysterious phenomena which we encountered while working in applied topology.

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