Tabulation of 3-manifolds of lengths up to 10

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Abstract

In previous work, we listed the prime links and the prime link exteriors with lengths up to 10. In this paper, we make a table of 3-manifolds with lengths up to 10 by using the list of the prime link groups.

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1. Introduction

In [14] a method of enumerating all of the links, the link groups and the closed connected orientable 3-manifolds is proposed. The idea is to introduce a well-order on the set of links by embedding it into a well-ordered set of

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lattice points. This well-order also naturally induces a well-order on the set of prime link groups and eventually induces a well-order on the set of closed connected orientable 3-manifolds. By using this method, the first 28, 26 and 26 lattice points of lengths up to 7 corresponding to the prime links, the prime link groups and the closed connected orientable 3-manifolds are respectively tabulated without any computer aid in [14]. We enlarged the table of the first 28 lattice points of lengths up to 7 corresponding to the prime links into that of the first 443 lattice points of lengths up to 10 in [17] and made the list of the first 399 lattice points of lengths up to 10 corresponding to the prime link exteriors in [19]. We, however find an omission in the tables of [17] and [19]. In this paper, we enumerate the first 444 lattice points of lengths up to 10 corresponding to the prime links and the first 400 lattice points of lengths up to 10 corresponding to the prime link exteriors, which are the correct versions, and we make the table of the first 346 lattice points of lengths up to 10 corresponding to the 3-manifolds together with the manifold data due to the third author using computations with all of Regina [4], Snappy [7], and Matveev's Recogniser [22]. [4]

In Section 2, we introduce a well-order on the set of links as follows. Since any link is represented as a closed braid, we can assign a lattice point to a closed braid and have an injection from the set of links to the set of lattice points after introducing a well-order on the set of lattice points. By using the well-order and the injection, we give a well-order on the set of links.

In Section 3, we explain a method of a tabulation of prime links, with respect to this order. First, we introduce a "normalized"subset of lattice points which represents, as a set of closed braids, a set of links including all the prime links. We enumerate the lattice points of lengths up to 10 in this set with respect to the given well-order. Omitting non-prime links and prime links which have already appeared in this well-order, we have a table of prime links. For this purpose, we introduce elementary transformations on the set of lattice points to have the following property. Namely, if one lattice point is transformed into the other by an elementary transformation, then the two corresponding closed braids are of the same link type. By this procedure, we can easily pick up most of the links which should be omitted.

In Section 4, we describe a method of a tabulation of the fundamental

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groups of prime links. First, we enumerate the exteriors of prime links with lengths up to 10. Omitting the link exteriors which have already appeared in the given well-order, we have a table of prime link exteriors with lengths up to 10. Next, we show that for two arbitrary prime link exteriors with lengths up to 10, their fundamental groups are not isomorphic to each other. This implies that our table of prime link exteriors with lengths up to 10 coincides the table of prime link groups with lengths up to 10.

In Section 5, we explain a method of a tabulation of 3-manifolds. First, we enumerate the 3-manifolds obtained by the 0-surgery along the links in the table of prime link groups with lengths up to 10. Omitting the manifolds which have already appeared in the given well-order, we have the table of 3-manifolds with lengths up to 10. To confirm that two manifolds are not homeomorphic to each other, we use Kirby-Melvin's version ([21]) of Witten's and Reshetikhin-Turaev's 3-manifold invariant as well as the Alexander invariants.

In the final section (Section 6), we make the list of lattice points corresponding to the 3-manifolds with lengths up to 10 together with the data on the prime links and prime link groups. Since we describe the link obtained by each lattice point as Conway's notation, in Appendix (added by the second author) we show pictures of two or more component links with 10 crossings in [5].

2. Definition of a well-order on the set of links

Let $\mathbf{Z}$ be the set of integers, and $\mathbf{Z}^n$ the product of n copies of $\mathbf{Z}$. We put

$$\mathbf{X} = \coprod_{n=1}^{\infty} \mathbf{Z}^n = \{(x_1, x_2, \dots, x_n) \,|\, x_i \in \mathbf{Z},\ n = 1, 2, \dots\}.$$

We call elements of $\mathbf{X}$ *lattice points*. For a lattice point $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathbf{X}$, we put $\ell(\mathbf{x}) = n$ and call it the *length* of $\mathbf{x}$. Let $|\mathbf{x}|$ and $|\mathbf{x}|_N$ be the lattice points determined from $\mathbf{x}$ by the following formulas:
$|\mathbf{x}| = (|x_1|, |x_2|, \dots, |x_n|)$ and $|\mathbf{x}|_N = (|x_{j_1}|, |x_{j_2}|, \dots, |x_{j_n}|)$,
where $|x_{j_1}| \leqq |x_{j_2}| \leqq \cdots \leqq |x_{j_n}|$ and $\{j_1, j_2, \dots, j_n\} = \{1, 2, \dots, n\}$.

We define a well-order (called a *canonical order* [14]) on $\mathbf{X}$ as follows:

Definition 2.1. We define a well-order on $\mathbf{Z}$ by $0 < 1 < -1 < 2 < -2 < 3 < -3 \cdots$, and for $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ we define $\mathbf{x} < \mathbf{y}$ if we have one of the following conditions (1)-(4):

(1) $\ell(\mathbf{x}) < \ell(\mathbf{y})$.
(2) $\ell(\mathbf{x}) = \ell(\mathbf{y})$ and $|\mathbf{x}|_N < |\mathbf{y}|_N$ by the lexicographic order on the natural number order.
(3) $|\mathbf{x}|_N = |\mathbf{y}|_N$ and $|\mathbf{x}| < |\mathbf{y}|$ by the lexicographic order on the natural number order.
(4) $|\mathbf{x}| = |\mathbf{y}|$ and $\mathbf{x} < \mathbf{y}$ by the lexicographic order on the well-order of $\mathbf{Z}$ defined above.

For $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbf{X}$, we put

$$\min|\mathbf{x}| = \min_{1 \leqq i \leqq n}|x_i| \quad \text{and} \quad \max|\mathbf{x}| = \max_{1 \leqq i \leqq n}|x_i|.$$

Let $\beta(\mathbf{x})$ be the $(\max|\mathbf{x}| + 1)$-string braid determined from $\mathbf{x}$ by the identity $\beta(\mathbf{x}) = \sigma_{|x_1|}^{\mathrm{sign}(x_1)}\sigma_{|x_2|}^{\mathrm{sign}(x_2)}\cdots\sigma_{|x_n|}^{\mathrm{sign}(x_n)}$, where we define $\sigma_{|0|}^{\mathrm{sign}(0)} = 1$. We note that $\max|\mathbf{x}| + 1$ is the minimum string number of the braid indicated by the right-hand side of the identity. Let $\mathrm{cl}\beta(\mathbf{x})$ be the closure of the braid $\beta(\mathbf{x})$. Let $\mathbf{L}$ be the set of all links modulo equivalence, where two links are *equivalent* if there is a (possibly orientation-reversing) homeomorphism sending one to the other. Then we have a map $\mathrm{cl}\beta : \mathbf{X} \to \mathbf{L}$ sending $\mathbf{x}$ to $\mathrm{cl}\beta(\mathbf{x})$. By Alexander's braiding theorem, the map $\mathrm{cl}\beta$ is surjective. For $L \in \mathbf{L}$, we define a map $\sigma : \mathbf{L} \to \mathbf{X}$ by $\sigma(L) = \min\{\mathbf{x} \in \mathbf{X} \,|\, \mathrm{cl}\beta(\mathbf{x}) = L\}$. Then σ is a right inverse of $\mathrm{cl}\beta$ and hence is injective. Now we have a well-order on $\mathbf{L}$ by the following definition:

Definition 2.2. For $L, L' \in \mathbf{L}$, we define $L < L'$ if $\sigma(L) < \sigma(L')$.

For a link $L \in \mathbf{L}$, we call $\ell(\sigma(L))$ the *length* of L.

3. A method of a tabulation of prime links

Let $\mathbf{L}^{\mathrm{p}}$ be the subset of $\mathbf{L}$ consisting of the prime links, where we consider that the 2-component trivial link is not prime. We use the injection σ for our method of a tabulation of $\mathbf{L}^{\mathrm{p}}$. For $k \in \mathbf{Z}$, let k^n and $-k^n$ be the lattice points determined by $k^n = (\underbrace{k, k, \ldots, k}_{n})$ and $-k^n = (-k)^n$, respectively.

For $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, $\mathbf{y} = (y_1, y_2, \ldots, y_m) \in \mathbf{X}$, let $\mathbf{x}^T$, $-\mathbf{x}$, $(\mathbf{x},\ \mathbf{y})$ and

$\delta(\mathbf{x})$ be the lattice points determined by the following formulas:

$$\begin{array}{rcl}
\mathbf{x}^T & = & (x_n, \ldots, x_2, x_1), \\
-\mathbf{x} & = & (-x_1, -x_2, \ldots, -x_n), \\
(\mathbf{x},\ \mathbf{y}) & = & (x_1, \ldots, x_n, y_1, \ldots, y_m), \\
\delta(\mathbf{x}) & = & (x'_1, x'_2, \ldots, x'_n), \\
\text{where} \quad x'_i & = & \begin{cases} \text{sign}(x_i)(\max|\mathbf{x}| + 1 - |x_i|) & (x_i \neq 0) \\ 0 & (x_i = 0). \end{cases}
\end{array}$$

A point of our argument on a tabulation of prime links is to define some transformations between lattice points. We make this definition as follows:

Definition 3.1. Let $\mathbf{x},\ \mathbf{y},\ \mathbf{z},\ \mathbf{w} \in \mathbf{X},\ k,\ l,\ n \in \mathbf{Z}$ with $n > 0$ and $\varepsilon = \pm 1$. An *elementary transformation* on lattice points is one of the following operations (1)–(12) and their inverses $(1)^-$–$(12)^-$.

(1) $(\mathbf{x},\ k,\ -k,\ \mathbf{y}) \to (\mathbf{x},\ \mathbf{y})$
(2) $(\mathbf{x},\ k,\ \mathbf{y}) \to (\mathbf{x},\ \mathbf{y})$, where $|k| > \max|\mathbf{x}|,\ \max|\mathbf{y}|$.
(3) $(\mathbf{x},\ k,\ l,\ \mathbf{y}) \to (\mathbf{x},\ l,\ k,\ \mathbf{y})$, where $|k| > |l| + 1$ or $|l| > |k| + 1$.
(4) $(\mathbf{x},\ \varepsilon k^n,\ k+1,\ k,\ \mathbf{y}) \to (\mathbf{x},\ k+1,\ k,\ \varepsilon(k+1)^n,\ \mathbf{y})$, where $k(k+1) \neq 0$.
(5) $(\mathbf{x},\ k,\ \varepsilon(k+1)^n,\ -k,\ \mathbf{y}) \to (\mathbf{x},\ -(k+1),\ \varepsilon k^n,\ k+1,\ \mathbf{y})$, where $k(k+1) \neq 0$.
(6) $(\mathbf{x},\ \mathbf{y}) \to (\mathbf{y},\ \mathbf{x})$
(7) $\mathbf{x} \to \mathbf{x}^T$
(8) $\mathbf{x} \to -\mathbf{x}$
(9) $\mathbf{x} \to \delta(\mathbf{x})$
(10) $(1^n,\ \mathbf{x},\ \varepsilon,\ \mathbf{y}) \to (1^n,\ \mathbf{y},\ \varepsilon,\ \mathbf{x})$, where $\min|\mathbf{x}| \geqq 2$ and $\min|\mathbf{y}| \geqq 2$.
(11) $(k^2,\ \mathbf{x},\ \mathbf{y},\ -k^2,\ \mathbf{z},\ \mathbf{w}) \to (-k^2,\ \mathbf{x},\ \mathbf{w}^T,\ k^2,\ \mathbf{z},\ \mathbf{y}^T)$, where $\max|\mathbf{x}| < k < \min|\mathbf{y}|$, $\max|\mathbf{z}| < k < \min|\mathbf{w}|$ and $\mathbf{x}, \mathbf{y}, \mathbf{z}$ or $\mathbf{w}$ may be empty.
(12) $(\mathbf{x},\ k,\ (k+1)^2,\ k,\ \mathbf{y}) \to (\mathbf{x},\ -k,\ -(k+1)^2,\ -k,\ \mathbf{y}^T)$, where $\max|\mathbf{x}| < k < \min|\mathbf{y}|$ and $\mathbf{x}$ or $\mathbf{y}$ may be empty.

A meaning of the transformations of Definition 3.1 is given by the following lemma (See [14, 17]):

Lemma 3.2. If a lattice point $\mathbf{x}$ is transformed into a lattice point $\mathbf{y}$ by an elementary transformation, then we have $\text{cl}\beta(\mathbf{x}) = \text{cl}\beta(\mathbf{y})$ (modulo a split union of a trivial link for (1), (2), (9)).

The outline of a tabulation of prime links is the following (See [14, 17] for the details): Let Δ be the subset of $\mathbf{X}$ consisting of $0,\ 1^m$ for $m \geqq 2$

and $(x_1, x_2, \ldots, x_n)$, where $n \geqq 4$, $x_1 = 1$, $1 \leqq |x_i| \leqq \frac{n}{2}$, $|x_n| \geqq 2$ and $\{|x_1|, |x_2|, \ldots, |x_n|\} = \{1, 2, \ldots, \max|\mathbf{x}|\}$. Then we have $\sharp\{\mathbf{y} \in \Delta \,|\, \mathbf{y} < \mathbf{x}\} < \infty$ for every $\mathbf{x} \in \Delta$ and have $\sigma(\mathbf{L}^{\mathrm{p}}) \subset \Delta$. [5] First, we enumerate the lattice points of Δ under the canonical order and then we omit $\mathbf{x} \in \Delta$ from the sequence if $\mathrm{cl}\beta(\mathbf{x})$ is a non-prime link or a link which has already appeared in the table of prime links. By using Lemma 3.2, we see that if $\mathbf{x}$ is transformed into a smaller one, then $\mathbf{x}$ must be removed from the sequence. We can find most of the omittable lattice points in this way. We show a table of 444 prime links with lengths up to 10 in Table 1:

$O < 2^2_1 < 3_1 < 4^2_1 < 4_1 < 5_1 < 5^2_1 < 6^2_1 < 5_2 < 6_2 < 6^3_3 < 6^3_1 < 6_3 < 6^3_2 < 6^2_3 < 7_1 <$
$6^2_2 < 7^2_1 < 7^2_7 < 7^2_8 < 7^2_4 < 7^2_2 < 7^2_5 < 7^2_6 < 6_1 < 7_6 < 7_7 < 7^3_1 < 8^2_1 < 7_3 < 8_2 < 8^3_7 <$
$8^3_8 < 8^3_1 < 8_{19} < 8_{20} < 8_5 < 7_5 < 8_7 < 8_{21} < 8_{10} < 8^3_9 < 8^3_5 < 8_{16} < 8_9 < 8^3_2 < 8_{17} <$
$8^3_6 < 8^3_{10} < 8^3_4 < 8_{18} < 7^2_3 < 8^2_5 < 8^2_{16} < 8^2_{15} < 8^2_9 < 8^2_8 < 8^2_{12} < 8^2_{13} < 8^2_7 < 8^2_{10} < 8^2_{11} <$
$8^4_3 < 8^4_2 < 8^4_1 < 8^2_{14} < 8_{12} < 9_1 < 8^2_2 < 9^2_1 < 9^2_{43} < 9^2_{44} < 9^2_{13} < 9^2_{49} < 9^2_{51} < 9^2_{19} < 9^2_{50} <$
$8^2_3 < 9^2_2 < 9^2_{52} < 9^2_{20} < 9^2_{55} < 9^2_{31} < 9^2_{53} < 9^2_{54} < 8^2_4 < 9^2_{23} < 9^2_{57} < 9^2_{35} < 9^2_{40} < 9^2_5 < 9^2_{14} <$
$9^2_{21} < 9^2_{34} < 9^2_{37} < 9^2_{59} < 9^2_{29} < 9^2_{39} < 9^2_{61} < 9^2_{41} < 9^2_{42} < 8_6 < 9_{11} < 9_{43} < 9_{44} < 9_{36} <$
$9_{42} < 7_2 < 8_{14} < 9_{26} < 8_4 < 8^3_3 < 9^3_6 < 9^3_{13} < 9^3_{14} < 9^3_2 < 9^3_{19} < 9^3_{18} < 9^3_8 < 9_{45} < 9_{32} <$
$9^3_{11} < 8_8 < 9_{20} < 9^3_1 < 7_4 < 8_{11} < 9_{27} < 8_{13} < 8_{15} < 9_{24} < 9_{30} < 9^3_{17} < 9^3_{16} < 9^3_{15} < 9^3_4 <$
$9^3_{10} < 9^3_{20} < 9^3_{12} < 9^3_{21} < 9_{33} < 9_{46} < 9_{34} < 9_{47} < 9_{31} < 9_{28} < 9_{40} < 9^2_{11} < 9_{17} < 9_{22} <$
$9^3_5 < 9^3_9 < 9_{29} < 9^2_{12} < 8^2_6 < 9^2_{25} < 10^2_1 < 9_3 < 10_2 < 10^3_{44} < 10^3_{45} < 10^3_1 < 10_{124} < 10_{126} <$
$10_{46} < 10_{125} < 10^3_{50} < 10^3_{51} < 10^3_7 < 9_6 < 10_5 < 10_{127} < 10_{47} < 10^3_{56} < 10^3_{27} < 10_{139} <$
$10_{143} < 9_9 < 10_{62} < 10_{141} < 10_{148} < 10_{85} < 10^3_A < 10^3_{52} < 10^3_{31} < 10_{155} < 10_{100} < 10_9 <$
$10^3_2 < 10^3_8 < 10_{149} < 10_{82} < 10^3_{58} < 10^3_{35} < 10^3_{62} < 10^3_B < 10^3_{19} < 9_{16} < 10_{64} < 10^3_{60} <$
$10^3_{38} < 10_{94} < 10_{161} < 10_{159} < 10_{106} < 10_{112} < 10^3_{64} < 10^3_{21} < 10^3_{73} < 10^3_{41} < 10_{116} <$
$10^3_{43} < 9^2_3 < 10^2_6 < 10^2_{133} < 10^2_{134} < 10^2_{38} < 10^2_{132} < 10^2_{141} < 10^2_{140} < 10^2_{46} < 10_{17} < 10_{48} <$
$10^3_{30} < 10_{91} < 10^3_{33} < 10_{152} < 10_{79} < 10_{157} < 10_{104} < 10_{99} < 10^3_{42} < 10_{118} < 10_{109} <$
$10_{123} < 9^2_7 < 10^2_{11} < 9^2_4 < 9^2_{27} < 10^2_{56} < 10^2_{136} < 10^2_{139} < 10^2_{44} < 10^2_{138} < 10^2_{169} < 10^2_{163} <$
$10^2_{76} < 10^2_{162} < 10^2_{135} < 10^2_{155} < 10^2_{88} < 9^2_{45} < 10^2_{128} < 9^2_{56} < 9^2_{47} < 10^2_{160} < 10^2_{154} < 10^2_{94} <$
$10^2_{124} < 10^2_{137} < 10^2_{98} < 10^2_{176} < 9^2_{58} < 10^2_{177} < 10^2_{110} < 9^2_6 < 10^2_{10} < 10^2_{42} < 10^2_{18} < 9^2_9 <$
$9^2_{48} < 9^2_{18} < 10^2_{125} < 10^2_{31} < 10^2_{63} < 9^2_{26} < 10^2_{41} < 9^2_8 < 10^4_{12} < 10^4_{11} < 10^4_{10} < 9^4_1 < 10^4_2 <$
$9^2_{22} < 9^2_{38} < 10^2_{90} < 9^2_{46} < 10^2_{129} < 10^2_{35} < 9^2_{16} < 10^4_A < 10^4_B < 10^4_C < 10^4_D < 10^4_6 < 10^2_{167} <$
$9^2_{60} < 9^2_{30} < 10^2_{74} < 10^2_{85} < 10^2_{170} < 10^2_{172} < 10^2_{107} < 10^2_{174} < 10^2_{156} < 10^2_{101} < 10^2_{179} <$
$10^2_{175} < 10^2_{118} < 10^2_{183} < 10^2_{93} < 9^2_{36} < 10^2_{142} < 10^2_{108} < 10^2_{117} < 10^2_{97} < 10^4_{15} < 10^4_{13} <$
$10^4_{19} < 10^4_8 < 10^4_{20} < 10^2_{165} < 10^2_{72} < 9^2_{17} < 10^4_{16} < 10^4_{14} < 10^4_{17} < 10^4_4 < 10^2_{114} < 10^2_{180} <$
$10^2_{181} < 10^2_{178} < 10^2_{120} < 10^2_{184} < 10^2_{19} < 9^2_{28} < 10^2_{64} < 9^2_{33} < 10^2_{81} < 10^2_{130} < 10^2_{131} <$

[5] In [14], Δ is defined as a further restricted subset of lattice points.

$10^2_{37} < 10^2_{168} < 10^2_{161} < 10^2_{75} < 10^4_{18} < 10^4_{7} < 10^2_{86} < 10^2_{119} < 10^2_{91} < 10^2_{17} < 10^2_{30} < 10^4_{5} <$
$9_{15} < 10_{29} < 10^3_{49} < 10^3_{48} < 10^3_{5} < 10^2_{26} < 9^2_{15} < 10^2_{36} < 10^2_{62} < 10^2_{43} < 10^2_{79} < 10^2_{149} <$
$10^2_{144} < 10^2_{143} < 10^2_{150} < 10^2_{145} < 10^2_{50} < 10^2_{48} < 10^2_{83} < 9^2_{32} < 10^2_{104} < 10^2_{A} < 10^2_{171} <$
$10^2_{106} < 10^2_{173} < 10^2_{87} < 10^2_{158} < 10^2_{115} < 10^2_{116} < 10^2_{109} < 10^2_{27} < 10^2_{45} < 10^2_{84} < 9^2_{24} <$
$10^2_{66} < 10^2_{151} < 10^2_{148} < 10^2_{147} < 10^2_{152} < 10^2_{54} < 10^2_{105} < 10^2_{96} < 10^4_{9} < 10^4_{21} < 10^2_{121} < 8_1 <$
$9_{21} < 10_{42} < 9_8 < 9_{25} < 10_{71} < 10^3_{29} < 10^3_{61} < 9_{14} < 8_3 < 9_{12} < 10_{44} < 10^3_{12} < 10_{43} <$
$10^2_{25} < 10^2_{34} < 10^2_{39} < 10^2_{55} < 10^2_{78} < 10^4_{1} < 10^2_{146} < 10^2_{52} < 10^2_{100} < 10^2_{103} < 10^2_{112} <$
$10_{41} < 9_{19} < 10_{137} < 10_{59} < 10_{136} < 10_{138} < 10_{54} < 10_{70} < 10^3_{6} < 9^3_{7} < 10^3_{53} < 10^3_{10} <$
$10^3_{54} < 10^3_{37} < 10^3_{59} < 10_{45} < 10^3_{13} < 10^3_{24} < 10_{88} < 10^3_{34} < 10^2_{23}$

Table 1

Here the links are indicated by using the order of the links Conway enumerated at the end of [5]. We find 7 omissions and 1 overlap $10^2_{178} = 10^2_{182}$ and we write the omissions as 10^2_A, 10^3_A, 10^3_B, 10^4_A, 10^4_B, 10^4_C and 10^4_D. We also find an omission $\sigma(10_{54}) = (1, -2, 1, 3, 2^3, -4, 3, -4)$ in [17]. For the above 444 links L, we show the corresponding lattice points $\sigma(L)$ at the end of this paper.

4. A method of a tabulation of prime link groups

Since a knot is determined by its exterior by the Gordon-Luecke Theorem [10] and a prime knot exterior is determined by its group by Whitten's Theorem [27], we classify the link groups of two or more component links. In [19] we have a table of prime link exteriors. First we review it. Next we show that the exterior table coincides with the group table.

Definition 4.1. For r-component links L and L' in S^3, their Alexander polynomials $\Delta_L(t_1, \dots, t_r)$ and $\Delta_{L'}(t_1, \dots, t_r)$ are equivalent if there is an isomorphism $\varphi : (t_1, \dots, t_r | t_i t_j = t_j t_i (i, j = 1, \dots, r)) \to (t_1, \dots, t_r | t_i t_j = t_j t_i (i, j = 1, \dots, r))$ satisfying $\Delta_{L'}(t_1, \dots, t_r) = \pm t_1^{\lambda_1} \cdots t_r^{\lambda_r} \Delta_L(\varphi(t_1), \dots, \varphi(t_r))$ for some integers $\lambda_i,\ i = 1, \dots, r$.

For a link L in S^3, let $E(L) = cl(S^3 - N(L))$ be its exterior, where $N(L)$ is a regular neighborhood of L. Since an isomorphism between link groups induces the automorphism of the group ring sending the Alexander ideal of the former onto that of the latter (see p. 104 of [6]), we have the following lemma.

Lemma 4.2. For links L and L' in S^3, if there is an isomorphism $\pi_1(E(L)) \to \pi_1(E(L'))$, then their Alexander polynomials are equivalent in the sense of Definition 4.1.

By using the above lemma, we divide the prime links into several groups, each of which consists of the links with the equivalent Alexander polynomials. For two or more component prime links with lengths up to 10, there are 42 groups consisting of two or more elememts. We show them in Table 2.

(1) $4_1^2 < 7_7^2 < 9_{43}^2 < 9_{59}^2$

(2) $6_1^2 < 9_{49}^2$

(3) $5_1^2 < 7_8^2 < 8_{15}^2 < 9_{47}^2 < 10_{174}^2 < 10_{173}^2$

(4) $6_3^2 < 8_{16}^2 < 9_{45}^2 < 10_{128}^2$

(5) $7_3^2 < 9_{46}^2$

(6) $9_{50}^2 < 10_{132}^2$

(7) $7_5^2 < 9_{48}^2 < 10_{130}^2$

(8) $7_2^2 < 9_{54}^2 < 10_{140}^2$

(9) $7_4^2 < 9_{44}^2 < 10_{162}^2 < 10_{124}^2$

(10) $10_{176}^2 < 10_{178}^2$

(11) $9_{57}^2 < 10_{167}^2$

(12) $7_6^2 < 9_{55}^2 < 9_{56}^2 < 10_{160}^2 < 10_{161}^2$

(13) $9_{58}^2 < 10_{168}^2$

(14) $8_{12}^2 < 8_{10}^2 < 10_{163}^2 < 10_{129}^2 < 10_{170}^2 < 10_{131}^2$

(15) $8_{13}^2 < 10_{154}^2$

(16) $8_{11}^2 < 10_{125}^2$

(17) $9_{27}^2 < 9_{15}^2$

(18) $9_{18}^2 < 9_{36}^2 < 10_{145}^2$

(19) $9_{33}^2 < 9_{32}^2$

(20) $9_{13}^2 < 10_{143}^2$

(21) $9_{31}^2 < 10_A^2$

(22) $10_{56}^2 < 10_{34}^2$

(23) $10_{76}^2 < 10_{78}^2$

(24) $10_{31}^2 < 10_{36}^2 < 10_{50}^2$

(25) $10_{81}^2 < 10_{83}^2 < 10_{104}^2$

(26) $10_{107}^2 < 10_{93}^2 < 10_{91}^2 < 10_{106}^2$

(27) $10_{84}^2 < 10_{105}^2$

(28) $10_{48}^2 < 10_{66}^2$

(29) $6_3^3 < 8_7^3 < 10_{44}^3$

(30) $6_1^3 < 8_8^3 < 9_{13}^3 < 9_{17}^3$

(31) $6_2^3 < 8_9^3 < 9_{19}^3 < 9_{18}^3 < 10_{61}^3$

(32) $7_1^3 < 9_{14}^3 < 10_{48}^3$

(33) $8_3^3 < 10_{49}^3$

(34) $8_1^3 < 10_{45}^3$

(35) $10_{58}^3 < 10_{59}^3$

(36) $8_5^3 < 10_{56}^3$

(37) $8_6^3 < 10_{60}^3$

(38) $8_3^4 < 10_A^4$

(39) $8_2^4 < 10_{12}^4 < 10_B^4 < 10_C^4$

(40) $10_{16}^4 < 10_{17}^4$

(41) $8_1^4 < 10_D^4$

(42) $10_{15}^4 < 10_{13}^4 < 10_{18}^4$

Table 2

We divide each group into several subgroups with the homeomorphic exteriors. We have the following results, shown later. For (2), (4), (5), (6), (7), (11), (13), (16), (29), (30), (32), (33), (34), (35), (38), (39), (40), (41) and (42), their exteriors are homeomorphic to each others. For the rest of the groups, we have the homeomorphism types in Table 3.

(1) $E(4_1^2) \cong E(7_7^2) \cong E(9_{43}^2),\ E(9_{59}^2)$

(3) $E(5_1^2) \cong E(7_8^2) \cong E(8_{15}^2) \cong E(9_{47}^2),\ E(10_{174}^2),\ E(10_{173}^2)$

(8) $E(7_2^2),\ E(9_{54}^2),\ E(10_{140}^2)$

(9) $E(7_4^2) \cong E(9_{44}^2) \cong E(10_{124}^2),\ E(10_{162}^2)$

(10) $E(10_{176}^2),\ E(10_{178}^2)$

(12) $E(7_6^2) \cong E(10_{160}^2) \cong E(10_{161}^2),\ E(9_{55}^2) \cong E(9_{56}^2)$

(14) $E(8_{12}^2) \cong E(10_{131}^2),\ E(8_{10}^2) \cong E(10_{129}^2),\ E(10_{163}^2),\ E(10_{170}^2)$

(15) $E(8_{13}^2),\ E(10_{154}^2)$

(17) $E(9_{27}^2)\ E(9_{15}^2)$

(18) $E(9_{18}^2),\ E(9_{36}^2),\ E(10_{145}^2)$

(19) $E(9_{33}^2),\ E(9_{32}^2)$

(20) $E(9_{13}^2),\ E(10_{143}^2)$

(21) $E(9_{31}^2),\ E(10_A^2)$

(22) $E(10^2_{56}),\ E(10^2_{34})$	(23) $E(10^2_{76}),\ E(10^2_{78})$
(24) $E(10^2_{31}),\ E(10^2_{36}),\ E(10^2_{50})$	(25) $E(10^2_{81}),\ E(10^2_{83}),\ E(10^2_{104})$
(26) $E(10^2_{107}),\ E(10^2_{93}),\ E(10^2_{91}),\ E(10^2_{106})$	(27) $E(10^2_{84}),\ E(10^2_{105})$
(28) $E(10^2_{48}),\ E(10^2_{66})$	(31) $E(6^3_2) \cong E(9^3_{18}),\ E(8^3_9) \cong E(9^3_{19}) \cong E(10^3_{61})$
(36) $E(8^3_5),\ E(10^3_{56})$	(37) $E(8^3_6),\ E(10^3_{60})$

Table 3

We show these results. For each group, we have $E(L) \cong E(L')$ by a composition of twist homeomorphisms along trivial components. Next we show that for each group, the classified exteriors are not homeomorphic to each others.

For (1), $E(4^2_1),\ E(7^2_7)$ and $E(9^2_{43})$ are Seifert manifolds since 4^2_1 is a torus link. On the other hand, 9^2_{59} is decomposed into two nontrivial tangles and so $E(9^2_{59})$ is not a Seifert manifold. We conclude that $E(4^2_1) \cong E(7^2_7) \cong E(9^2_{43}) \not\cong E(9^2_{59})$.

For (3), let $5^2_1 = K_1 \cup K_2$ and $10^2_{174} = K'_1 \cup K'_2$, where $K_1 = K_2 = K'_1 = O$ and $K'_2 = 6_2$. Suppose that there is a homeomorphism $h : E(5^2_1) \to E(10^2_{174})$. We may assume $h(\partial N(K_1)) = \partial N(K'_1)$. Let $K'_{2,n}$ be a knot obtaind by twisting K'_2 along K'_1 n times. Since $\mathrm{lk}(K_1, K_2) = \mathrm{lk}(K'_1, K'_2) = 0$, $E(K_2)$ should be homeomorphic to $E(K'_{2,n})$ for some integer n and then $K_2 = K'_{2,n}$. However this is impossible. So we have $E(5^2_1) \not\cong E(10^2_{174})$. The same argument implies that $E(5^2_1) \not\cong E(10^2_{173})$ and $E(10^2_{174}) \not\cong E(10^2_{173})$. We conclude that there are 3 homeomorphism types of exteriors: $E(5^2_1) \cong E(7^2_8) \cong E(8^2_{15}) \cong E(9^2_{47})$, $E(10^2_{174})$ and $E(10^2_{173})$.

For (9), (12), (14), (15), (17), (18), (19), (20), (21), (22), (23), (24), (25), (26) and (27), we have the result in the same way as in (3).

For (8), suppose that there is a homeomorphism $h : E(7^2_2) \to E(9^2_{54})$. From their Alexander polynomials $\Delta_{7^2_2}(t_1, t_2),\ \Delta_{9^2_{54}}(t_1, t_2)$, we see $h_*(t_1) = t_i^{\pm 1}$ and $h_*(t_2) = t_j^{\pm 1}$ where $h_* : H_1(E(7^2_2)) \to H_1(E(9^2_{54}))$ is an isomorphism induced by h, t_1 and t_2 are meridians of the homology groups, and $\{i, j\} = \{1, 2\}$. Since the linking numbers for 7^2_2 and 9^2_{54} are both non-zero, the homeomorphism h preserves the meridians of $E(7^2_2)$ and $E(9^2_{54})$ and extends to a homeomorphism from S^3 to S^3 sending 7^2_2 to 9^2_{54}, which is impossible. So we have $E(7^2_2) \not\cong E(9^2_{54})$. By the same methods, we also have $E(7^2_2) \not\cong E(10^2_{140})$ and $E(9^2_{54}) \not\cong E(10^2_{140})$.

For (10), we compute the first homology groups of the double covering spaces. Those of $E(10^2_{176})$ are $\mathbf{Z}^3 \oplus \mathbf{Z}_7,\ \mathbf{Z}^3,\ \mathbf{Z}^3$, and those of $E(10^2_{178})$ are $\mathbf{Z}^3 \oplus \mathbf{Z}_9,\ \mathbf{Z}^3,\ \mathbf{Z}^3$, where we write $A^n = \underbrace{A \oplus \cdots \oplus A}_{n}$ for an abelian group A.

For (28), as in (10), we have the first homology groups of the double covering spaces of $E(10^2_{48})$ are $\mathbf{Z}^2 \oplus \mathbf{Z}_{36},\ \mathbf{Z}^3,\ \mathbf{Z}^3$ and those of $E(10^2_{66})$ are $\mathbf{Z}^2 \oplus \mathbf{Z}_3 \oplus \mathbf{Z}_{12},\ \mathbf{Z}^3,\ \mathbf{Z}^3$.

For (31), the absolute values of the linking numbers of all pairs of the components for 6^3_2 are $0, 0, 0$ and those for 8^3_9 are $0, 0, 2$ and we have $E(6^3_2) \not\cong E(8^3_9)$.

For (36), the absolute values of the linking numbers of all pairs of the components for 8^3_5 are $0, 0, 1$ and those for 10^3_{56} are $0, 0, 3$ and we have $E(8^3_5) \not\cong E(10^3_{56})$.

For (37), we compute the first homology groups of the double covering spaces. Those of $E(8^3_6)$ are $\mathbf{Z}^3 \oplus (\mathbf{Z}_3)^2,\ \mathbf{Z}^4 \oplus \mathbf{Z}_3,\ \mathbf{Z}^3,\ \mathbf{Z}^3,\ \mathbf{Z}^3,\ \mathbf{Z}^4,\ \mathbf{Z}^4$ and those of $E(10^3_{60})$ are $\mathbf{Z}^3 \oplus \mathbf{Z}_9,\ \mathbf{Z}^4 \oplus \mathbf{Z}_3,\ \mathbf{Z}^3,\ \mathbf{Z}^3,\ \mathbf{Z}^3,\ \mathbf{Z}^4,\ \mathbf{Z}^4$.

Next we show that in each group, $E(L) \not\cong E(L')$ induces $\pi_1(E(L)) \not\cong \pi_1(E(L'))$. In the following arguments, we refer to Jaco's book [11] as a general reference of 3-manifold topology (see also Appendix C of [13]).

For a prime link L, the exterior $E(L)$ is *simple* if there is no essential torus in $E(L)$. Then we also call the link L a *simple* link. By the torus theorem and by a classification result of a Seifert link by G. Burde-K. Murasugi [3], we have the following well-known lemma.

Lemma 4.3. Every simple link exterior $E(L)$ is a hyperbolic 3-manifold or a special Seifert manifold with the orbit space a punctured sphere.

Assume that $E(L)$ is a special Seifert manifold. Then the Seifert structure of $E(L)$ comes from a Seifert structure on S^3 by a classification result of G. Burde-K. Murasugi [3]. By [11], the orbit surface of the special Seifert manifold $E(L)$ is in one of the following cases.

(i) the disk with at most two exceptional fibers,
(ii) the annulus with at most one exceptional fiber, and
(iii) the disk with two holes and no exceptional fibers.

We denote the torus link of type (p, q) by $K_{p,q}$ for non-negative integers p, q. In the case (i), the link L is a torus knot $K_{p,q}$ for coprime positive integers p, q. In the case (ii), the type of the exceptional fiber of $E(L)$ is given by a pair (p, q) for coprime positive integers p, q. Then we can take L as the union $L_{p,q} = S^1 \cup K_{p,q}$ for the torus knot $K_{p,q}$ and an axis S^1 of the Seifert fiberation of S^3 with two exceptional fibers of types (p, q) and (q, p), where the notation $L_{p,q}$ is taken to have the *linking degree* $|lk(S^1, K_{p,q})| = p$. The link $L_{1,q}$ is a Hopf link and $L_{p,1}$ is the torus link $K_{2,2p}$. For any coprime integers p, q greater than 1, the link $L_{p,q}$ is not a torus link since it contains two distinct knot components. In the case (iii), $E(L)$ is homeomorphic to $S^1 \times D(2)$ for the disk $D(2)$ with two holes and L is taken as $L_{2,0}$. We note that the exterior $E(L_{2,q})$ of the link $L_{2,q}$ for an even positive integer q is also a special Seifert manifold, but it is homeomorphic to $E(L_{2,0})$. We have the following lemma, which is a corrected version of Proposition 4.6 in [14] which was incorrectly stated although the error did not have any influence on the overall result of [14].

Lemma 4.4. Assume that L is a prime link and L_0 is a simple link. The fundamental group $\pi_1(E(L))$ is isomorphic to $\pi_1(E(L_0))$ if and only if $E(L)$ is homeomorphic to $E(L_0)$ or the pair (L_0, L) is equivalent to a pair $(L_{p,q}\ ,\ L_{p,q'})$ for some positive integers p, q, q' with $(p, q) = (p, q') = 1$. Further, $E(L_{p,q})$ is homeomorphic to $E(L_{p,q'})$ if and only if $q' \equiv \pm q \pmod{p}$. Thus, if L and L_0 have crossing numbers smaller than 15, then $\pi_1(E(L))$ is isomorphic to $\pi_1(E(L_0))$ if and only if $E(L)$ is homeomorphic to $E(L_0)$.

Proof. By Thurston's hyperbolization theorem, the exterior $E(L_0)$ is hyperbolic if and only if every subgroup $\mathbf{Z} \oplus \mathbf{Z}$ of $\pi_1(E(L_0))$ is peripheral and $E(L_0)$ is not homeomorphic to the twisted line bundle over the Klein bottle. Since $E(L_0)$ is a link exterior, $E(L_0)$ is not homeomorphic to the twisted line bundle over the Klein bottle. Thus, $E(L_0)$ is a hyperbolic manifold if and only if every subgroup $\mathbf{Z} \oplus \mathbf{Z}$ of $\pi_1(E(L_0))$ is peripheral. Assume that $E(L_0)$ is hyperbolic and there is an isomorphism $\phi : \pi_1(E(L)) \to \pi_1(E(L_0))$. Since every peripheral subgroup $\mathbf{Z} \oplus \mathbf{Z}$ of $\pi_1(E(L))$ is sent to a peripheral subgroup of $\pi_1(E(L_0))$ by ϕ, namely ϕ preserves a peripheral structure, we see that $E(L)$ is homeomorphic to $E(L_0)$.

Next, assume that $E(L_0)$ is a special Seifert manifold and there is an isomorphism $\phi : \pi_1(E(L)) \to \pi_1(E(L_0))$. By [3], $E(L)$ is also a special Seifert fibered manifold with the same orbit data as $E(L_0)$. Thus, if $E(L_0)$ is in the case (i), then L is the same torus knot as L_0 since the fundamental group determines the torus knot type. If $E(L_0)$ is in the case (iii), then we have $E(L) \cong E(L_0) = S^1 \times D(2)$.

Assume that $E(L_0)$ is in the case (ii). Let (p', q') and (p, q) be the types of the exceptional fibers of $E(L)$ and $E(L_0)$, respectively. The fundamental groups have the following presentations:

$$\pi_1(E(L)) = (t, a, b | ta = at, tb = bt, t^{q'} = a^{p'}) \quad \text{and}$$
$$\pi_1(E(L_0)) = (t, a, b | ta = at, tb = bt, t^{q} = a^{p}).$$

These presentations of $E(L)$ and $E(L_0)$ are respectively obtained from

$S^1 \times D(2)$ by adjoining a fibered solid torus around the exceptional fiber. Let $\phi : \pi_1(E(L)) \to \pi_1(E(L_0))$ be an isomorphism. Considering the central group C which is the infinite cyclic group generated by t, we see that $\phi(t) = t^{\pm 1}$. In the quotient groups by C, ϕ induces an isomorphism $\phi_* : (a|a^{p'} = 1) * (b|-) \cong (a|a^p = 1) * (b|-)$. Hence we have $p' = p$. Thus, there are homeomorphisms $E(L) \cong E(L_{p,q'})$ and $E(L_0) \cong E(L_{p,q})$ for some positive integeres p, q, q' with $(p, q) = (p, q') = 1$. We show that there is an isomorphism $\pi_1(E(L_{p,q'})) \to \pi_1(E(L_{p,q}))$. Since q and q' are units in the cyclic group $\mathbf{Z}/p\mathbf{Z}$, we can find integers r and s such that $q' = qr + ps$ and $(r, p) = 1$. We define a homomorphism $\eta : (t, a, b|-) \to \pi_1(E(L_{p,q}))$ for the free group $(t, a, b|-)$ with generators t, a and b by $\eta(t) = t$, $\eta(a) = a^r t^s$ and $\eta(b) = b$. Then we have

$$\eta(ta) = ta^r t^s = a^r t^s t = \eta(at),\ \eta(tb) = tb = bt = \eta(bt),$$
$$\eta(t^{q'} a^{-p}) = t^{q'}(a^r t^s)^{-p} = (t^q a^{-p})^r = 1.$$

This means that η induces a homomorphism $\phi = \eta_\# : \pi_1(E(L_{p,q'})) \to \pi_1(E(L_{p,q}))$. To see that ϕ is injective, let $\phi(x) = 1$ for an element x in $\pi_1(E(L_{p,q'}))$. Because $(p, r) = 1$, the homomorphism ϕ induces an isomorphism $\phi_* : (a|a^p = 1) * (b|-) \cong (a|a^p = 1) * (b|-)$ in the quotient groups by the central subgroup C generated by t. Thus, x is the identity element in $\pi_1(E(L_{p,q'}))/C$ and hence $x = t^k$ for an integer k. Then $\phi(x) = t^k = 1$ means $k = 0$. Thus, ϕ is injective. Next, we show that ϕ is onto, let y be an element in $\pi_1(E(L_{p,q}))$. Using the isomorphism ϕ_*, we have an element x in $\pi_1(E(L_{p,q'}))$ such that $\phi(x) = yt^m$ for an integer m. Then $\phi(xt^{-m}) = y$ and hence ϕ is onto. Thus, we showed that ϕ is an isomorphism. If there is a homeomorphism $E(L_{p,q'}) \to E(L_{p,q})$, then there is a fiber-preserving homeomorphism $E(L_{p.q'}) \to E(L_{p,q})$ by Orlik, Vogt and Zieschang [25] (cf. [3]) and hence $q' \equiv \pm q \pmod{p}$.

To see the last assertion of the lemma, it suffices to observe that the smallest crossing number example is given by $E(L_{5,1})$ and $E(L_{5,2})$ where the link $L_{5,1}$ has crossing number 10 because it is a torus link $K_{2,10}$ and the link $L_{5,2}$ has crossing number 15 because $|lk(S^1, K_{5,2})| = 5$ and the torus knot $K_{5,2}$ has 5 crossings. □

We note that $L_{p,1}$ is a 2-bridge link and $L_{p,2}$ is a 3-bridge link. By taking p as any odd integer greater than or equal to 5, we obtain the following corollary from Lemma 4.4.

Corollary 4.5. There are infinitely many distinct link pairs consisting of a prime 2-bridge link and a prime 3-bridge link whose fundamental groups are isomorphic.

The following lemma presents some examples of simple links which are useful for our study, where we use the prime links L'_m and L''_m illustrated in Fig. 1.

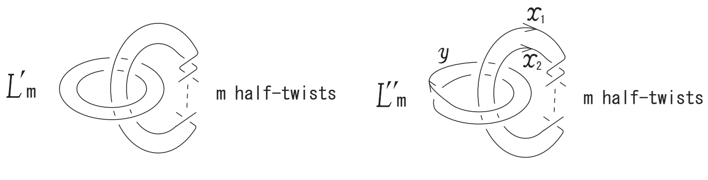

Fig. 1

Lemma 4.6. Every prime link L belonging to the following classes (1)-(5) is simple.
(1) The prime 2-bridge links or more generally the prime links whose double branched covering spaces are simple 3-manifolds.
(2) The prime pretzel links $P(d_1, d_2, \ldots, d_n)$ ($d_i \in \mathbf{Z}$, $i = 1, 2, \ldots, n$) or more generally the prime Montesinos links $M(r_1, r_2, \ldots, r_n)$ ($r_i \in \mathbf{Q}$, $i = 1, 2, \ldots, n$) except for the following prime non-simple links.

$$\begin{aligned} L'_0 &= P(2,2,-2,-2) = M(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{-1}{2}, \tfrac{-1}{2}),\\ L''_{-1} &= P(1,-3,-3,-3) = M(\tfrac{2}{3}, \tfrac{-1}{3}, \tfrac{-1}{3}),\\ L''_{-2} &= P(2,-4,-4) = M(\tfrac{1}{2}, \tfrac{-1}{4}, \tfrac{-1}{4}),\\ L''_{-3} &= P(2,-3,-6) = M(\tfrac{1}{2}, \tfrac{-1}{3}, \tfrac{-1}{6}). \end{aligned}$$

(3) The prime alternating links.
(4) The prime 3-bridge knots.
(5) The prime links with a trivial component such that infinitely many Dehn twists along it make the remaining sublinks simple links.

Proof. Let L be in (1). Let T be a torus in $E(L)$, whose lifting $\widetilde{T}$ to the double branched covering space $S(L)_2$ is homeomorphic to one torus or two tori. We note that if L is a prime 2-bridge link, then $S(L)_2$ is a lens space except S^3 and $S^1 \times S^2$. Since $\widetilde{T}$ is compressible in $S(L)_2$, we find an equivariant disk for $\widetilde{T}$ in $S(L)_2$ by the equivariant Dehn's lemma. Since L is prime, we see that T is compressible or boundary-parallel in $E(L)$, showing that $E(L)$ is simple. Since every prime 2-bridge link is an alternating link, the conclusion for 2-bridge links will be also obtained from (3).

(2) is shown by Oertel [24], where we note that the exterior of a prime Montesinos link is a Seifert fiber space if and only if it is a special Seifert manifold because such Montesinos links are classified as $L_{n,1} = K_{2,2n}$ ($n \geqq 1$), $L_{2,n}$ ($n \geqq 2$), $L_{3,2}$, $K_{3,4}$ or $K_{3,5}$ (see Bonahon-Seibenmann [2]).

(3) is shown by Menasco [23], where we note that every prime alternating link is a hyperbolic link or a torus link $K_{2,q}$ for an integer $q \geqq 2$.

To see (4), let L be a 3-bridge knot. If there is an essential torus T in $E(L)$, then we take a solid torus V bounded by T in S^3, whose core knot is denoted by K. Since T is essential, K is a non-trivial knot and L is a satellite of K with winding number, say w. Then we have the inequality $3 = b(L) \geqq wb(K)$ for the bridge numbers $b(L)$ and $b(K)$ by H. Schubert [26]. Since $b(K) \geqq 2$, we have $w = 1$ and L must be a non-trivial connected sum, contradicting that L is a prime knot. This shows (4).

To see (5), let L be in (5) and suppose that $E(L)$ is not simple. Then we consider a torus decomposition of $E(L)$. Let T_0 be a torus component of $\partial E(L)$ around the trivial knot component K_0 of L. Let E' be a Seifert or hyperbolic piece containing T_0. Let E' be a Seifert piece. Twistings of the sublink $L \backslash K_0$ along K_0 induce Dehn fillings E'' of E' along T_0. Thus, except a finite number of twists along K_0, any twist of $L \backslash K_0$ along K_0 makes E'' a Seifert or hyperbolic manifold by a well-known argument of Dehn surgery. The exterior of the link L^* obtained from $L \backslash K_0$ by this twist along K_0 has an essential torus, a contradiction. □

By Jaco's result in [11, p. 212], if the exterior $E(L_0)$ of a prime link L_0 has no essential annulus, then we see that the exterior $E(L)$ of a prime link L is homeomorphic to $E(L_0)$ if and only if there is an isomorphism $\pi_1(E(L)) \to \pi_1(E(L_0))$. We have further examples.

Theorem 4.7. Let a prime link L_0 be a 2-bridge link, an alternating link, a pretzel link or a Montesinos link except the torus link $K_{2,2p} = L_{p,1}$ for any integer $p \geqq 5$. Then the exterior $E(L)$ of a prime link L is homeomorphic to $E(L_0)$ if and only if there is an isomorphism $\pi_1(E(L)) \to \pi_1(E(L_0))$.

Proof. Assume that $L_0 \neq L'_0, L''_{-1}, L''_{-2}, L''_{-3}$. Then L_0 is a simple link by Lemma 4.6. Further, if $L_0 \neq L_{p,1}$ ($p \geqq 5$), then we have $L_0 \neq L_{p,q}$ for any coprime positive integers p, q with $p \geqq 5$ by the argument of Lemma 4.6, so that $E(L_0) \neq E(L_{p,q})$ by Lemma 4.4. Thus, in this case, we see from Lemma 4.4 that the exterior $E(L)$ of a prime link L is homeomorphic to $E(L_0)$ if and only if $\pi_1(E(L))$ is isomorphic to $\pi_1(E(L_0))$.

Let L_0 be one of $L'_0, L''_{-1}, L''_{-2}, L''_{-3}$. By Fig. 1, $E(L_0)$ is a torus sum of two simple pieces, namely two copies of $E(L_{2,0})$ for $L_0 = L'_0$, $E(L_{2,0})$ and $E(L_{2,1})$ for $L_0 = L''_{-2}$, or two copies of $E(L_{2,1})$ for $L_0 = L''_{-1}, L''_{-3}$. Let L be a prime link such that there is an isomorphism $\phi : \pi_1(E(L)) \to \pi_1(E(L_0))$. Then by Johannson's theorem (see Jaco [11, p. 212])), $E(L)$ is also a torus sum of the same pieces as the pieces of the torus decomposition of $E(L_0)$ because the heomeomorphism types of these pieces are determined by the fundamental groups. If the 3-sphere S^3 is obtained as a union of two solid tori V_j $(j = 1, 2)$ with unoriented meridian-longitude systems given, then the meridian loop of V_1 must be attached to a longitude of V_2 obtained from the given longitude of V_2 by twisting several times along the meridian disk of V_2. This means that the link L is equivalent to the link L'_m or L''_m for an integer m illustrated in Fig. 1 where m denotes the m half-twists.

Let $L_0 = L'_0$. Then we have $L = L'_m$ for an even integer m and we can see that $E(L) = E(L'_m)$ is homeomorphic to $E(L'_0) = E(L_0)$ by the $-\frac{m}{2}$ full-twists along a component of L'_m.

Let $L_0 = L''_{-1}$. Then $L = L''_m$ for an odd integer m. The Alexander polynomial of L''_m is calculated as $\Delta(L''_m) = (1 + x^2y)(x^m + y^2)$ by using y and x represented by x_1 and x_2 in Fig. 1. Since $\pi_1(E(L''_m))$ is isomorphic to $\pi_1(E(L''_{-1}))$, we have

$$(1 + x^2y)(x^m + y^2) = \pm x^\lambda y^\mu(1 + \psi(x)^2\psi(y))(\psi(x)^{-1} + \psi(y)^2)$$

for integers λ, μ and a transformation ψ on x, y such that $\psi(x) = x^ay^b$ and $\psi(y) = x^cy^d$ for integers a, b, c, d with $ad - bc = \pm 1$ (see Lemma 4.2). Then we have $m = -1, -7$. Since we see that $E(L''_{-7})$ is (orientation-reversingly) homeomorphic to $E(L''_{-1})$ by considering the mirror image of L''_{-7}, it follows that $E(L)$ is homeomorphic to $E(L''_{-1}) = E(L_0)$ (see Fig. 2).

Let $L_0 = L''_{-2}$. Then we have $L = L''_m$ for an even integer m. Let $m = 2m'$. The Alexander polynomial of L''_m is calculated as

$$\Delta(L''_m) = (1 + x_1x_2y)((x_1x_2)^{m'} - y^2)$$

by using x_1, x_2 and y in Fig. 1. By Johannson's theorem in [11], any homotopy equivalence $E(L''_m) \to E(L''_{-2})$ preseves the characteristic torus of the torus decompositions of $E(L''_m)$ and $E(L''_{-2})$. Since x_1, x_2 and y represent a basis of the first homology of the characteristic torus in $E(L_0)$ and $E(L)$, we have

$$(1+x_1x_2y)((x_1x_2)^{m'} - y^2) = \pm x_1^{\lambda_1}x_2^{\lambda_2}y^\mu(1+\psi(x_1x_2)\psi(y))(\psi(x_1x_2)^{-1} - \psi(y)^2)$$

for integers λ_1, λ_2, μ and a transformation ψ on x_1, x_2, y such that $\psi(x_1x_2) = (x_1x_2)^ay^b$ and $\psi(y) = (x_1x_2)^cy^d$ for integers a, b, c, d with $ad - bc = \pm 1$.

Then we have $m' = -1, -3$ and $m = -2, -6$. Since we see that $E(L''_{-6})$ is (orientation-reversingly) homeomorphic to $E(L''_{-2})$ by considering the mirror image of L''_{-6}, it follows that $E(L)$ is homeomorphic to $E(L''_{-2}) = E(L_0)$ (see Fig. 2).

Let $L_0 = L''_{-3}$. Then $L = L''_m$ for an odd integer m and the Alexander polynomial of L''_m is $\Delta(L''_m) = (1+x^2y)(x^m+y^2)$ by using y and x represented by x_1 and x_2 in Fig. 1. Since $\pi_1(E(L''_m))$ is isomorphic to $\pi_1(E(L''_{-3}))$, we have $(1 + x^2y)(x^m + y^2) = \pm x^\lambda y^\mu (1 + \psi(x)^2\psi(y))(\psi(x)^{-3} + \psi(y)^2)$ for integers λ, μ and a transformation ψ such that $\psi(x) = x^a y^b$ and $\psi(y) = x^c y^d$ for integers a, b, c, d with $ad-bc = \pm 1$. Then we have $m = -3, -5$. Since we see that $E(L''_{-5})$ is (orientation-reversingly) homeomorphic to $E(L''_{-3})$ by considering the mirror image of L''_{-5}, it follows that $E(L)$ is homeomorphic to $E(L''_{-3}) = E(L_0)$ (see Fig. 2). $\square$

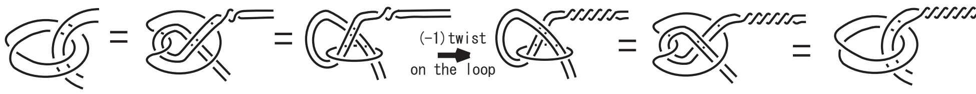

Fig. 2

Now we have prepared to show that in each group of Table 3, $E(L) \not\cong E(L')$ induces $\pi_1(E(L)) \not\cong \pi_1(E(L'))$.

For (1), since 4^2_1 is a prime alternating link, 4^2_1 is a simple link by Lemma 4.6 and $E(4^2_1) \not\cong E(9^2_{59})$ induces $\pi_1(E(4^2_1)) \not\cong \pi_1(E(9^2_{59}))$ by Lemma 4.4. The same method works for (9),(12),(15),(17),(18),(19),(20),(21),(22),(23),(24), (25),(26),(27),(31),(36),(37) and we see that in each group, $E(L) \not\cong E(L')$ induces $\pi_1(E(L)) \not\cong \pi_1(E(L'))$.

For (3), since 5^2_1 is a prime alternating link and 10^2_{174} has a trivial component such that infinitely many Dehn twists along it make the other component a prime 3-bridge knot, 5^2_1 and 10^2_{174} are simple links and we have any two of $\pi_1(E(5^2_1))$, $\pi_1(E(10^2_{174}))$ and $\pi_1(E(10^2_{173}))$ are not isomorphic to each other. For (14), we have the result in the same way as in (3).

For (8), since 7^2_2 is a prime alternating link, 7^2_2 is a simple link and we have $\pi_1(E(7^2_2)) \not\cong \pi_1(E(9^2_{54}))$ and $\pi_1(E(7^2_2)) \not\cong \pi_1(E(10^2_{140}))$. To show that $\pi_1(E(9^2_{54})) \not\cong \pi_1(E(10^2_{140}))$, we compute the first homology groups of the double covering spaces. Those of $E(9^2_{54})$ are $\mathbf{Z}^2 \oplus (\mathbf{Z}_3)^2$, $\mathbf{Z}^2$, $\mathbf{Z}^2$, and those of $E(10^2_{140})$ are $\mathbf{Z}^2 \oplus \mathbf{Z}_9$, $\mathbf{Z}^2$, $\mathbf{Z}^2$. So we have $\pi_1(E(9^2_{54})) \not\cong \pi_1(E(10^2_{140}))$.

For (10) and (28), as we mentioned, by computing the first homology groups of the double covering spaces, we have $\pi_1(E(10^2_{176})) \not\cong \pi_1(E(10^2_{178}))$ and $\pi_1(E(10^2_{48})) \not\cong \pi_1(E(10^2_{66}))$.

We obtain a table of the lattice points of lengths up to 10 corresponding to prime link groups, by omitting the lattice points corresponding to

$$7^2_7, 7^2_8, 8^3_7, 8^3_8, 8^2_{16}, 8^2_{15}, 9^2_{43}, 9^2_{44}, 9^2_{49}, 9^3_{13}, 9^3_{14}, 9^3_{19}, 9^3_{18}, 9^3_{17}, 10^3_{44}, 10^3_{45}, 10^2_{132}, 9^2_{45}, 10^2_{128}, 9^2_{56}, 9^2_{47}, 10^2_{160}, 10^2_{124}, 9^2_{48}, 10^2_{125}, 10^4_{12}, 9^2_{46}, 10^2_{129}, 10^4_A, 10^4_B, 10^4_C, 10^4_D, 10^2_{167}, 10^4_{13}, 10^4_{17}, 10^2_{130}, 10^2_{131}, 10^2_{168}, 10^2_{161}, 10^4_{18}, 10^3_{49}, 10^3_{48}, 10^3_{61}, 10^3_{59}$$

from Table 1 because their link groups have already appeared. Thus, we have the table of 400 prime link groups listed in Table 4:

$O < 2^2_1 < 3_1 < 4^2_1 < 4_1 < 5_1 < 5^2_1 < 6^2_1 < 5_2 < 6_2 < 6^3_3 < 6^3_1 < 6_3 < 6^3_2 < 6^2_3 < 7_1 < 6^2_2 < 7^2_1 < 7^2_4 < 7^2_2 < 7^2_5 < 7^2_6 < 6_1 < 7_6 < 7_7 < 7^3_1 < 8^2_1 < 7_3 < 8_2 < 8^3_1 < 8_{19} < 8_{20} < 8_5 < 7_5 < 8_7 < 8_{21} < 8_{10} < 8^3_9 < 8^3_5 < 8_{16} < 8_9 < 8^3_2 < 8_{17} < 8^3_6 < 8^3_{10} < 8^3_4 < 8_{18} < 7^2_3 < 8^2_5 < 8^2_9 < 8^2_8 < 8^2_{12} < 8^2_{13} < 8^2_7 < 8^2_{10} < 8^2_{11} < 8^4_3 < 8^4_2 < 8^4_1 < 8^2_{14} < 8_{12} < 9_1 < 8^2_2 < 9^2_1 < 9^2_{13} < 9^2_{51} < 9^2_{19} < 9^2_{50} < 8^2_3 < 9^2_2 < 9^2_{52} < 9^2_{20} < 9^2_{55} < 9^2_{31} < 9^2_{53} < 9^2_{54} < 8^2_4 < 9^2_{23} < 9^2_{57} < 9^2_{35} < 9^2_{40} < 9^2_5 < 9^2_{14} < 9^2_{21} < 9^2_{34} < 9^2_{37} < 9^2_{59} < 9^2_{29} < 9^2_{39} < 9^2_{61} < 9^2_{41} < 9^2_{42} < 8_6 < 9_{11} < 9_{43} < 9_{44} < 9_{36} < 9_{42} < 7_2 < 8_{14} < 9_{26} < 8_4 < 8^3_3 < 9^3_6 < 9^3_2 < 9^3_8 < 9_{45} < 9_{32} < 9^3_{11} < 8_8 < 9_{20} < 9^3_1 < 7_4 < 8_{11} < 9_{27} < 8_{13} < 8_{15} < 9_{24} < 9_{30} < 9^3_{16} < 9^3_{15} < 9^3_4 < 9^3_{10} < 9^3_{20} < 9^3_{12} < 9^3_{21} < 9_{33} < 9_{46} < 9_{34} < 9_{47} < 9_{31} < 9_{28} < 9_{40} < 9^2_{11} < 9_{17} < 9_{22} < 9^3_5 < 9^3_9 < 9_{29} < 9^2_{12} < 8^2_6 < 9^2_{25} < 10^2_1 < 9_3 < 10_2 < 10^3_1 < 10_{124} < 10_{126} < 10_{46} < 10_{125} < 10^3_{50} < 10^3_{51} < 10^3_7 < 9_6 < 10_5 < 10_{127} < 10_{47} < 10^3_{56} < 10^3_{27} < 10_{139} < 10_{143} < 9_9 < 10_{62} < 10_{141} < 10_{148} < 10_{85} < 10^3_A < 10^3_{52} < 10^3_{31} < 10_{155} < 10_{100} < 10_9 < 10^3_2 < 10^3_8 < 10_{149} < 10_{82} < 10^3_{58} < 10^3_{35} < 10^3_{62} < 10^3_B < 10^3_{19} < 9_{16} < 10_{64} < 10^3_{60} < 10^3_{38} < 10_{94} < 10_{161} < 10_{159} < 10_{106} < 10_{112} < 10^3_{64} < 10^3_{21} < 10^3_{73} < 10^3_{41} < 10_{116} < 10^3_{43} < 9^2_3 < 10^2_6 < 10^2_{133} < 10^2_{134} < 10^2_{38} < 10^2_{141} < 10^2_{140} < 10^2_{46} < 10_{17} < 10_{48} < 10^3_{30} < 10_{91} < 10^3_{33} < 10_{152} < 10_{79} < 10_{157} < 10_{104} < 10_{99} < 10^3_{42} < 10_{118} < 10_{109} < 10_{123} < 9^2_7 < 10^2_{11} < 9^2_4 < 9^2_{27} < 10^2_{56} < 10^2_{136} < 10^2_{139} < 10^2_{44} < 10^2_{138} < 10^2_{169} < 10^2_{163} < 10^2_{76} < 10^2_{162} < 10^2_{135} < 10^2_{155} < 10^2_{88} < 10^2_{154} < 10^2_{94} < 10^2_{137} < 10^2_{98} < 10^2_{176} < 9^2_{58} < 10^2_{177} < 10^2_{110} < 9^2_6 < 10^2_{10} < 10^2_{42} < 10^2_{18} < 9^2_9 < 9^2_{18} < 10^2_{31} < 10^2_{63} < 9^2_{26} < 10^2_{41} < 9^2_8 < 10^4_{11} < 10^4_{10} < 9^4_1 < 10^4_2 < 9^2_{22} < 9^2_{38} < 10^2_{90} < 10^2_{35} < 9^2_{16} < 10^4_6 < 9^2_{60} < 9^2_{30} < 10^2_{74} < 10^2_{85} < 10^2_{170} < 10^2_{172} < 10^2_{107} < 10^2_{174} < 10^2_{156} < 10^2_{101} < 10^2_{179} < 10^2_{175} < 10^2_{118} < 10^2_{183} < 10^2_{93} < 9^2_{36} < 10^2_{142} < 10^2_{108} < 10^2_{117} < 10^2_{97} < 10^4_{15} < 10^4_{19} < 10^4_8 < 10^4_{20} < 10^2_{165} < 10^2_{72} < 9^2_{17} < 10^4_{16} < 10^4_{14} < 10^4_4 < 10^2_{114} < 10^2_{180} < 10^2_{181} < 10^2_{178} < 10^2_{120} < 10^2_{184} < 10^2_{19} < 9^2_{28} < 10^2_{64} < 9^2_{33} < 10^2_{81} < 10^2_{37} < 10^2_{75} < 10^4_7 < 10^2_{86} < 10^2_{119} < 10^2_{91} < 10^2_{17} < 10^2_{30} < 10^4_5 < 9_{15} < 10_{29} < 10^3_5 < 10^2_{26} < 9^2_{15} < 10^2_{36} < 10^2_{62} < 10^2_{43} < 10^2_{79} < 10^2_{149} < 10^2_{144} < 10^2_{143} < 10^2_{150} < 10^2_{145} < 10^2_{50} < 10^2_{48} < 10^2_{83} < 9^2_{32} < 10^2_{104} < 10^2_A < 10^2_{171} < 10^2_{106} < 10^2_{173} < 10^2_{87} < 10^2_{158} < 10^2_{115} < 10^2_{116} < 10^2_{109} < 10^2_{27} < 10^2_{45} < 10^2_{84} < 9^2_{24} < 10^2_{66} < 10^2_{151} < 10^2_{148} < 10^2_{147} < 10^2_{152} < 10^2_{54} < 10^2_{105} < 10^2_{96} < 10^4_9 < 10^4_{21} < 10^2_{121} < 8_1 < 9_{21} < 10_{42} < 9_8 < 9_{25} < 10_{71} < 10^3_{29} < 9_{14} < 8_3 < 9_{12} < 10_{44} < 10^3_{12} < 10_{43} < 10^2_{25} < 10^2_{34} < 10^2_{39} < 10^2_{55} < 10^2_{78} < 10^4_1 < 10^2_{146} < 10^2_{52} < 10^2_{100} < 10^2_{103} < 10^2_{112} < 10_{41} < 9_{19} < 10_{137} < 10_{59} <$

$10_{136} < 10_{138} < 10_{54} < 10_{70} < 10^3_6 < 9^3_7 < 10^3_{53} < 10^3_{10} < 10^3_{54} < 10^3_{37} < 10_{45} < 10^3_{13} < 10^3_{24} < 10_{88} < 10^3_{34} < 10^2_{23}$

Table 4

5. A method of a tabulation of 3-manifolds

We make a list of closed connected orientable 3-manifolds by constructing a sequence of 3-manifolds obtained by the 0-surgery of the links in Table 4 and removing the manifolds which have already appeared. Thus, we have the table of 3-manifolds of lengths up to 10 which will be listed in Section 6. If we continue this procedure, then all closed connected orientable 3-manifolds are tabulated (see [14]). Let $\chi(L, 0)$ denote the manifold obtained by the 0-surgery of a link L. We classify $\chi(L, 0)$ for L in Table 4 according to the first homology group $H_1(\chi(L, 0))$. There are 16 types of groups 0, $\mathbf{Z}$, $\mathbf{Z}\oplus\mathbf{Z}$, $\mathbf{Z}\oplus\mathbf{Z}\oplus\mathbf{Z}$, $\mathbf{Z}\oplus\mathbf{Z}\oplus\mathbf{Z}\oplus\mathbf{Z}$, $\mathbf{Z}\oplus\mathbf{Z}_2$, $\mathbf{Z}\oplus\mathbf{Z}_2\oplus\mathbf{Z}_2$, $\mathbf{Z}\oplus\mathbf{Z}_3\oplus\mathbf{Z}_3$, $\mathbf{Z}_2$, $\mathbf{Z}_4$, $\mathbf{Z}_6$, $\mathbf{Z}_8$, $\mathbf{Z}_2\oplus\mathbf{Z}_2$, $\mathbf{Z}_3\oplus\mathbf{Z}_3$, $\mathbf{Z}_4\oplus\mathbf{Z}_4$, $\mathbf{Z}_5\oplus\mathbf{Z}_5$ and we have respectively 50, 141, 69, 6, 1, 4, 6, 1, 12, 7, 5, 7, 60, 21, 9, 1 links with these types of groups.

Case 1. $H_1(\chi(L, 0)) \cong 0$.

We enumerate the manifolds $\chi(L, 0)$ with $H_1(\chi(L, 0)) \cong 0$. The links with this condition in our group table are the following:

$2^2_1 < 7^2_1 < 7^2_2 < 8^2_8 < 8^2_7 < 9^2_{19} < 9^2_{50} < 9^2_{52} < 9^2_{54} < 9^2_{35} < 9^2_{21} < 9^2_{34} < 9^2_{39} < 9^2_{42} < 9^2_{11} < 9^2_{12} < 9^2_3 < 10^2_{134} < 10^2_{140} < 10^2_{136} < 10^2_{139} < 10^2_{44} < 10^2_{138} < 10^2_{88} < 10^2_{137} < 10^2_{98} < 10^2_{42} < 10^2_{41} < 9^2_8 < 10^4_2 < 10^2_{85} < 10^2_{101} < 10^2_{118} < 10^2_{183} < 10^2_{117} < 10^2_{114} < 10^2_{180} < 10^2_{64} < 10^2_{86} < 10^2_{119} < 10^2_{62} < 10^2_{43} < 10^2_{87} < 10^2_{115} < 10^2_{109} < 10^2_{45} < 10^2_{121} < 10^2_{39} < 10^4_1 < 10^2_{112}$.

We see that $\chi(9^2_{54}, 0) \cong \chi(10^2_{134}, 0)$, $\chi(9^2_{50}, 0) \cong \chi(10^2_{140}, 0)$, $\chi(7^2_2, 0) \cong \chi(10^4_2, 0)$, $\chi(7^2_1, 0) \cong \chi(10^4_1, 0)$ and remove 10^2_{134}, 10^2_{140}, 10^4_2, 10^4_1 from our sequence. The first two homeomorphisms can be obtained as follows: they are Mazur manifolds of types $\chi(9^2_{54}, 0) \cong W^+(0, -1)$, $\chi(10^2_{134}, 0) \cong W^+(-1, 0)$, $\chi(9^2_{50}, 0) \cong W^+(1, 0)$, $\chi(10^2_{140}, 0) \cong W^+(0, 1)$ and we have $W^+(0, -1) \cong W^+(-1, 0)$ and $W^+(1, 0) \cong W^+(0, 1)$ by Proposition 1 in [1]. The last two homeomorphisms are obtained by using handle slides in Fig. 3.

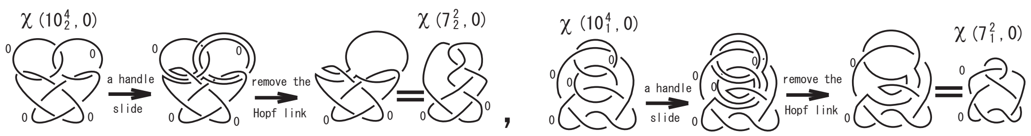

Fig. 3

For the rest of the links L, we calculate $\tau_5(\chi(L,0))$, Kirby-Melvin's version ([21]) of Witten's and Reshetikhin-Turaev's 3-manifold invariant.

We introduce the invariant quickly. If r is a fixed integer greater than 1 and M is described by surgery on an oriented framed link L, then $\tau_r(M)$ is defined by the following formula:

$$\tau_r(M) = \alpha \sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{r-1}} [\mathbf{k}] J_{L,\mathbf{k}},$$

where $\alpha = b^n c^\sigma,\ b = \sqrt{\frac{2}{r}} \sin\frac{\pi}{r},\ c = e^{\frac{-3(r-2)\pi i}{4r}}$, n is the number of components of L, σ is the signature of the linking matrix of L, $\mathbf{k} = (k_1, \dots, k_n)$ is a coloring of $L = K_1 \cup \cdots \cup K_n$ assigning an integer k_i to K_i, $\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{r-1}}$ is the sum over all $\mathbf{k}$ with $1 \leqq k_i \leqq r-1$, $[\mathbf{k}] = \prod_{i=1}^n [k_i]$, $[k_i] = \frac{s^{k_i} - \bar{s}^{k_i}}{s - \bar{s}},\ s = e^{\frac{\pi i}{r}}$,

$$J_{L,\mathbf{k}} = [2] \sum_{\mathbf{j}=\mathbf{0}}^{\frac{\mathbf{k-1}}{2}} (-1)^{\mathbf{j}} \binom{\mathbf{k-1-j}}{\mathbf{j}} t^{3L^{\mathbf{k-1-2j}} \cdot L^{\mathbf{k-1-2j}}} V_{L^{\mathbf{k-1-2j}}}(\bar{q}) \ ,$$

$\sum_{\mathbf{j}=\mathbf{0}}^{\frac{\mathbf{k-1}}{\mathbf{2}}}$ is the sum over all $\mathbf{j} = (j_1, \dots, j_n)$ with $0 \leqq j_i \leqq \frac{k_i - 1}{2}$,

$$(-1)^{\mathbf{j}} = \prod_{i=1}^n (-1)^{j_i},\ \binom{\mathbf{k}}{\mathbf{j}} = \prod_{i=1}^n \binom{k_i}{j_i},\ t = e^{\frac{\pi i}{2r}},\ L^{\mathbf{k}} = K_1^{k_1} \cup \cdots \cup K_n^{k_n},$$

$K_i^{k_i}$ is k_i parallel pushoffs using the framing, $K_i^0 = \emptyset$, $L \cdot L = \sum_{i=1}^n \sum_{j=1}^n K_i \cdot K_j$, $K_i \cdot K_i$ is the framing of K_i and $K_i \cdot K_j = \mathrm{lk}(K_i, K_j)$ for $i \neq j$, $V_L(x)$ is the Jones polynomial of L, $q = e^{\frac{2\pi i}{r}}$ and $q^{\frac{1}{2}}$ is chosen to be $-s$.

They show that if two manifolds M and M' are homeomorphic to each other by an orientation preserving [resp. reversing] homeomorphism, $\tau_r(M) = \tau_r(M')$ [resp. $\tau_r(M) = \overline{\tau_r(M')}$].

We apply the fact to our situation and have the following lemma.

Lemma 5.1. Let $L = K_1 \cup K_2$ be an oriented 2-component link with the linking number l, then $\tau_5(\chi(L,0))$ and $\tau_7(\chi(L,0))$ are computed by the following formulas:

$\tau_5(\chi(L,0)) = \frac{2}{5}\left(\sin^2\frac{\pi}{5}\right)(3 + (-1)^l)(1 + [2]^2 f_L),$

$f_L = V_{K_1}(\bar{q})+V_{K_2}(\bar{q})+[2]\, q^{\frac{3l}{2}}\, V_L(\bar{q}),\ [2] = s^{-1}+s,\ s = e^{\frac{\pi i}{5}},\ q = e^{\frac{2\pi i}{5}},\ q^{\frac{1}{2}} = -s.$

$\tau_7(\chi(L,0)) = \frac{2}{7}\left(\sin^2\frac{\pi}{7}\right)(3+(-1)^l)(1-2[3]+[3]^2+[2]f_L),$

$$f_L = [2](1-[3])(V_{K_1}(\bar{q})+V_{K_2}(\bar{q}))+(-1)^l[2]^2s^{3l}V_L(\bar{q}) \\ +[3](1-[3])(V_{K_1^2}(\bar{q})+V_{K_2^2}(\bar{q}))+[2][3]s^{6l}(V_{K_1^2\cup K_2}(\bar{q})+V_{K_1\cup K_2^2}(\bar{q})) \\ +[3]^2s^{12l}V_{L^2}(\bar{q}),$$

$[2] = s^{-1}+s,\ [3] = s^{-2}+1+s^2,\ s = e^{\frac{\pi i}{7}},\ q = e^{\frac{2\pi i}{7}},\ q^{\frac{1}{2}} = -s.$

In Case 1, we have $\tau_5(\chi(L,0)) = C(1+(s^{-2}+2+s^2)\cdot f_L)$, where $C = \frac{4}{5}\sin^2\frac{\pi}{5}$ and $s = e^{\frac{\pi i}{5}}$. We show the values of f_L for the rest of the links L in Table 5.

L	f_L	L	f_L
2^2_1	1	7^2_1	$2+3s-2s^2+s^3$
7^2_2	$2s+2s^2-s^3$	8^2_8	$1+5s^2-5s^3$
8^2_7	$1+5s-5s^2$	9^2_{19}	$3-4s+s^2+2s^3$
9^2_{50}	$2s-3s^2-s^3$	9^2_{52}	$-3+3s-2s^2+s^3$
9^2_{54}	$-3s+2s^2-s^3$	9^2_{35}	$7s-8s^2+4s^3$
9^2_{21}	$-2+6s-4s^2+2s^3$	9^2_{34}	$-3+3s+3s^2-4s^3$
9^2_{39}	$-3+8s-7s^2+s^3$	9^2_{42}	$-4+5s+5s^2-5s^3$
9^2_{11}	$-3+8s-7s^2+s^3$	9^2_{12}	$-3s+7s^2-6s^3$
9^2_3	$-2+6s-4s^2+2s^3$	10^2_{136}	$2-2s+3s^2+s^3$
10^2_{139}	$-4+5s$	10^2_{44}	$-4+10s-10s^2+5s^3$
10^2_{138}	$2s-3s^2+4s^3$	10^2_{88}	$-3+13s-12s^2+6s^3$
10^2_{137}	$1-5s+5s^2$	10^2_{98}	$-3+13s-12s^2+6s^3$
10^2_{42}	$6-5s+5s^3$	10^2_{41}	$-4+5s^2-5s^3$
9^2_8	$-1+4s-s^2-2s^3$	10^2_{85}	$-4+10s^2-10s^3$
10^2_{101}	$-5+2s+7s^2-6s^3$	10^2_{118}	$-9+15s-10s^2$
10^2_{183}	$1-5s+5s^2-5s^3$	10^2_{117}	$-4+10s^2-10s^3$
10^2_{114}	$-5+2s+12s^2-11s^3$	10^2_{180}	$1+5s-5s^2+5s^3$
10^2_{64}	$-4+10s-10s^2+5s^3$	10^2_{86}	$-4+15s-15s^2+5s^3$
10^2_{119}	$-3+18s-17s^2+6s^3$	10^2_{62}	$1-5s+10s^2-5s^3$
10^2_{43}	$-8s+12s^2-6s^3$	10^2_{87}	$2-7s+13s^2-9s^3$
10^2_{115}	$1-10s+15s^2-10s^3$	10^2_{109}	$1-10s+15s^2-10s^3$
10^2_{45}	$-8+13s-7s^2+s^3$	10^2_{121}	$-4+15s^2-15s^3$
10^2_{39}	$6-5s+5s^3$	10^2_{112}	$7-7s-2s^2+6s^3$

Table 5

We conclude that the manifolds are different from each other except the following 11 cases:

$\tau_5(\chi(9^2_{50},0)) \equiv \tau_5(\chi(10^2_{136},0),\ \tau_5(\chi(9^2_{52},0)) \equiv \tau_5(\chi(9^2_{54},0)),$
$\tau_5(\chi(9^2_{21},0)) \equiv \tau_5(\chi(9^2_3,0)),\ \tau_5(\chi(9^2_{34},0)) \equiv \tau_5(\chi(9^2_{12},0)),$
$\tau_5(\chi(9^2_{39},0)) \equiv \tau_5(\chi(9^2_{11},0)), \tau_5(\chi(10^2_{139},0)) \equiv \tau_5(\chi(10^2_{183},0)),$
$\tau_5(\chi(10^2_{44},0)) \equiv \tau_5(\chi(10^2_{64},0)),\ \tau_5(\chi(10^2_{88},0)) \equiv \tau_5(\chi(10^2_{98},0)),$

$\tau_5(\chi(10^2_{42},0)) \equiv \tau_5(\chi(10^2_{39},0)),\ \tau_5(\chi(10^2_{85},0)) \equiv \tau_5(\chi(10^2_{117},0)),$
$\tau_5(\chi(10^2_{115},0)) \equiv \tau_5(\chi(10^2_{109},0)),$

where $\alpha \equiv \beta$ means $\alpha = \beta$ or $\alpha = \bar{\beta}$ for complex values α, β and we say α is equivalent to β. For these manifolds, we compute $\tau_7(\chi(L,0))$ in [21], which is described as $\tau_7(\chi(L,0)) = C(1-2[3]+[3]^2+[2]\cdot f_L)$, where $C = \frac{4}{7}\sin^2\frac{\pi}{7}$, $[2] = s^{-1}+s$, $[3] = s^{-2}+1+s^2$, $s = e^{\frac{\pi i}{7}}$, and f_L is a complex number determined by L. We show the values of f_L for these undetermined manifolds in Table 6.

L	f_L	L	f_L
9^2_{50}	$-2-s+s^2+4s^3+s^5$	10^2_{136}	$4+s+5s^2+s^3+2s^4-7s^5$
9^2_{52}	$-3+s-2s^2+s^3+2s^4$	9^2_{54}	$5-s+8s^2-3s^3-6s^5$
9^2_{21}	$2s-7s^2+3s^3-4s^4+3s^5$	9^2_{3}	$7+2s+3s^3+3s^4-4s^5$
9^2_{34}	$-3+8s+5s^2-6s^3+9s^4-14s^5$	9^2_{12}	$-2-8s+15s^2-17s^3+7s^4-6s^5$
9^2_{39}	$-3+8s-16s^2+15s^3-12s^4+7s^5$	9^2_{11}	$4+8s-9s^2+15s^3-5s^4$
10^2_{139}	$1+3s^2-8s^3+8s^4-10s^5$	10^2_{183}	$1-7s+10s^2-8s^3+s^4-3s^5$
10^2_{44}	$8-4s^2+13s^3-13s^4+11s^5$	10^2_{64}	$1-11s^2+13s^3-20s^4+11s^5$
10^2_{88}	$11-6s-9s^2+22s^3-26s^4+14s^5$	10^2_{98}	$18-6s-2s^2+15s^3-26s^4+14s^5$
10^2_{42}	$-6+10s^2-15s^3+15s^4-17s^5$	10^2_{39}	$1+14s-11s^2+13s^3+s^4-10s^5$
10^2_{85}	$-13+24s^2-43s^3+43s^4-31s^5$	10^2_{117}	$1+38s^2-50s^3+50s^4-38s^5$
10^2_{115}	$22-42s+59s^2-43s^3+15s^4-3s^5$	10^2_{109}	$15-49s+52s^2-50s^3+15s^4-3s^5$

Table 6

Since these values of f_L are not equivalent to each other, we have the enumeration of 3-manifolds in Case 1.

Case 2. $H_1(\chi(L,0)) \cong \mathbf{Z}$.

We enumerate the manifolds $\chi(L,0)$ with $H_1(\chi(L,0)) \cong \mathbf{Z}$. The links in our group table with this condition are the following:

$O < 3_1 < 4_1 < 5_1 < 5_2 < 6_2 < 6_3 < 7_1 < 6_1 < 7_6 < 7_7 < 7_3 < 8_2 < 8_{19} < 8_{20} < 8_5 < 7_5 < 8_7 < 8_{21} < 8_{10} < 8^3_5 < 8_{16} < 8_9 < 8_{17} < 8^3_6 < 8_{18} < 8_{12} < 9_1 < 8_6 < 9_{11} < 9_{43} < 9_{44} < 9_{36} < 9_{42} < 7_2 < 8_{14} < 9_{26} < 8_4 < 9^3_2 < 9_{45} < 9_{32} < 8_8 < 9_{20} < 9^3_1 < 7_4 < 8_{11} < 9_{27} < 8_{13} < 8_{15} < 9_{24} < 9_{30} < 9^3_{10} < 9_{33} < 9_{46} < 9_{34} < 9_{47} < 9_{31} < 9_{28} < 9_{40} < 9_{17} < 9_{22} < 9_{29} < 9_3 < 10_2 < 10_{124} < 10_{126} < 10_{46} < 10_{125} < 9_6 < 10_5 < 10_{127} < 10_{47} < 10_{139} < 10_{143} < 9_9 < 10_{62} < 10_{141} < 10_{148} < 10_{85} < 10^3_A < 10^3_{31} < 10_{155} < 10_{100} < 10_9 < 10_{149} < 10_{82} < 10^3_{62} < 10^3_B < 10^3_{19} < 9_{16} < 10_{64} < 10^3_{60} < 10^3_{38} < 10_{94} < 10_{161} < 10_{159} < 10_{106} < 10_{112} < 10_{116} < 10^3_{43} < 10_{17} < 10_{48} < 10^3_{30} < 10_{91} < 10_{152} < 10_{79} < 10_{157} < 10_{104} < 10_{99} < 10^3_{42} < 10_{118} < 10_{109} < 10_{123} < 9_{15} < 10_{29} < 8_1 < 9_{21} < 10_{42} < 9_8 < 9_{25} < 10_{71} < 10^3_{29} < 9_{14} < 8_3 < 9_{12} < 10_{44} < 10^3_{12} < 10_{43} < 10_{41} < 9_{19} < 10_{137} < 10_{59} < 10_{136} < 10_{138} < 10_{54} < 10_{70} < 10^3_{37} < 10_{45} < 10^3_{13} < 10_{88} < 10^3_{34}.$

The 3-manifolds obtained by the 0-surgery of the above three component links are homeomorphic to those of some knots as in Table 7, where 3_1^* is the mirror image of 3_1 and we show the pictures of $K_1, K_2, \ldots, K_7$ in Fig. 4.

$$\chi(8^3_5,0) \cong \chi(9_{46},0), \quad \chi(8^3_6,0) \cong \chi(3_1\sharp 3^*_1,0), \quad \chi(9^3_2,0) \cong \chi(6_3,0),$$
$$\chi(9^3_1,0) \cong \chi(6_2,0), \quad \chi(9^3_{10},0) \cong \chi(K_1,0), \quad \chi(10^3_A,0) \cong \chi(8_{20},0),$$
$$\chi(10^3_{31},0) \cong \chi(K_2,0), \quad \chi(10^3_{62},0) \cong \chi(O,0), \quad \chi(10^3_B,0) \cong \chi(10_{140},0),$$
$$\chi(10^3_{19},0) \cong \chi(K_3,0), \quad \chi(10^3_{60},0) \cong \chi(6_1,0), \quad \chi(10^3_{38},0) \cong \chi(10_{153},0),$$
$$\chi(10^3_{43},0) \cong \chi(K_4,0), \quad \chi(10^3_{30},0) \cong \chi(K_5,0), \quad \chi(10^3_{42},0) \cong \chi(K_6,0),$$
$$\chi(10^3_{29},0) \cong \chi(K_7,0), \quad \chi(10^3_{12},0) \cong \chi(7_6,0), \quad \chi(10^3_{37},0) \cong \chi(3^*_1\sharp 5_2,0),$$
$$\chi(10^3_{13},0) \cong \chi(7_7,0), \quad \chi(10^3_{34},0) \cong \chi(4_1\sharp 4_1,0).$$

Table 7

So, we omit 9^3_2, 9^3_1, 9_{46}, 10^3_A, 10^3_{62}, 10^3_{60}, 10^3_{12}, 10^3_{13} from the sequence.

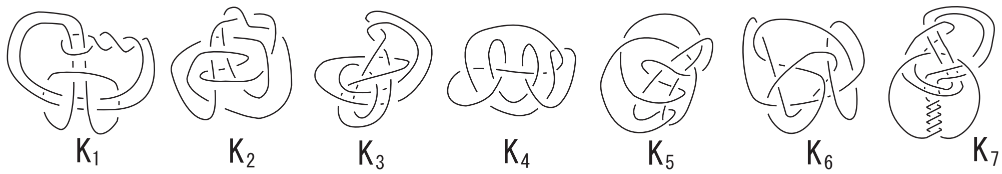

Fig. 4

For the rest of the links, we can see, by calculating the Alexander polynomials, that the manifolds are different from each other except the cases shown in Table 8.

$\Delta(\chi(L,0))$	L	$\Delta(\chi(L,0))$	L
1	$O < 9^3_{10} < 10^3_{19}$	$3+0-1$	$10^3_{31} < 10^3_{30}$
$3-2+1$	$8_{20} < 8^3_6 < 10^3_B < 10^3_{43}$	$5-2$	$6_1 < 8^3_5 < 10^3_{42} < 10^3_{29}$
$5-4+1$	$8_{21} < 10_{136}$	$5-4+3-1$	$8_5 < 10_{141}$
$7-5+3-1$	$8_9 < 10_{155}$	$7-6+3-1$	$8_{10} < 10_{143}$
$11-6+1$	$10_{137} < 10^3_{34}$	$11-8+2$	$8_{14} < 9_8$
$11-9+5-1$	$9_{20} < 10_{149}$	$13-10+5-1$	$8_{18} < 9_{24}$
$15-12+5-1$	$9_{28} < 9_{29}$	$23-18+7-1$	$9_{40} < 10_{59}$

Table 8

In Table 8, the Alexander polynomial $\Delta(\chi(L,0)) = a_0 + a_1(t+t^{-1}) + \cdots + a_n(t^n + t^{-n})$ is abbreviated by $a_0 + a_1 + \cdots + a_n$. For the above links, we replace the 0-framed three component links with the 0-framed knots, as we described before, and we substitute the fifth root of unity $q = e^{\frac{2\pi i}{5}}$ for the Jones polynomials of the knots, and we have Table 9.

$L \to K$	$V_K(q)$	$L \to K$	$V_K(q)$
O	1	$9^3_{10} \to K_1$	$3q - 3q^2 + q^3$
$10^3_{19} \to K_3$	$2 + 2q - 2q^2 + 4q^3$	$10^3_{31} \to K_2$	$-3 - q - q^2 - 4q^3$
$10^3_{30} \to K_5$	$5 + 3q^2 + 3q^3$	8_{20}	$2 + q + 3q^3$
$8^3_6 \to 3_1 \sharp 3^*_1$	$4 + q^2 + q^3$	$10^3_B \to 10_{140}$	$2q - q^2$
$10^3_{43} \to K_4$	$1 + 4q - 3q^2 + 4q^3$	6_1	$4 + 2q + 2q^2 + 3q^3$
$8^3_5 \to 9_{46}$	$1 + q - 2q^2 + q^3$	$10^3_{42} \to K_6$	$8 + 4q^2 + 4q^3$
$10^3_{29} \to K_7$	$-3 + 3q - 4q^2$	8_{21}	$2 - 3q + 3q^2 - q^3$
10_{136}	$-5 - 2q - 3q^2 - 4q^3$	8_5	$-5 + q - 4q^2 - q^3$
10_{141}	$3 + 6q^2 + 2q^3$	8_9	$8 + 4q^2 + 4q^3$
10_{155}	$5 + 4q + 7q^3$	8_{10}	$-5 - 6q - 8q^3$
10_{143}	$3 - 5q + 4q^2 - q^3$	10_{137}	$5 + 4q + 7q^3$
$10^3_{34} \to 4_1 \sharp 4_1$	$8 + 4q^2 + 4q^3$	8_{14}	$1 - 5q + 5q^2 - 5q^3$
9_8	$-6 - 4q - q^2 - 8q^3$	9_{20}	$-10 + 2q - 6q^2 - 5q^3$
10_{149}	$-13 - 4q - 5q^2 - 12q^3$	8_{18}	$15 + 8q^2 + 8q^3$
9_{24}	$13 + q + 7q^2 + 10q^3$	9_{28}	$-5 - 10q + 3q^2 - 12q^3$
9_{29}	$-3 + 9q - 6q^2 + 6q^3$	9_{40}	$-7 - 13q + 5q^2 - 19q^3$
10_{59}	$1 + 13q - 6q^2 + 13q^3$		

Table 9

If $\chi(K,0) \cong \chi(K',0)$, then we have $V_K(q) = V_{K'}(q)$ or $\overline{V_{K'}(q)}$ by Kirby and Melvin's theorem [21], p. 530. So the above calculations show that all the manifolds are different from each other.

Case 3. $H_1(\chi(L,0)) \cong \mathbf{Z} \oplus \mathbf{Z}$.

We enumerate the manifolds with $H_1(\chi(L,0)) \cong \mathbf{Z} \oplus \mathbf{Z}$. The links with this condition in our group table are the following:

$5^2_1 < 7^2_4 < 7^2_6 < 7^2_3 < 8^2_{12} < 8^2_{13} < 8^2_{10} < 8^4_3 < 8^4_1 < 9^2_{13} < 9^2_{55} < 9^2_{31} < 9^2_5 < 9^2_{37} < 9^2_{41} < 9^2_{25} < 9^2_4 < 9^2_{27} < 10^2_{56} < 10^2_{169} < 10^2_{163} < 10^2_{76} < 10^2_{162} < 10^2_{154} < 10^2_{94} < 10^2_{176} < 9^2_9 < 9^2_{18} < 10^2_{31} < 10^2_{170} < 10^2_{107} < 10^2_{174} < 10^2_{93} < 9^2_{36} < 10^2_{178} < 10^2_{120} < 9^2_{33} < 10^2_{81} < 10^4_7 < 10^2_{91} < 10^2_{26} < 9^2_{15} < 10^2_{36} < 10^2_{79} < 10^2_{143} < 10^2_{150} < 10^2_{145} < 10^2_{50} < 10^2_{48} < 10^2_{83} < 9^2_{32} < 10^2_{104} < 10^2_A < 10^2_{106} < 10^2_{173} < 10^2_{158} < 10^2_{116} < 10^2_{27} < 10^2_{84} < 10^2_{66} < 10^2_{148} < 10^2_{152} < 10^2_{105} < 10^4_9 < 10^2_{25} < 10^2_{34} < 10^2_{78} < 10^2_{146} < 10^2_{103}$.

The following homeomorphisms

$$\chi(5^2_1,0) \cong \chi(8^4_1,0),\ \chi(7^2_6,0) \cong \chi(10^4_7,0),\ \chi(8^4_3,0) \cong \chi(10^4_9,0)(\cong \chi(0^2_1,0))$$

are obtained by using handle slides. The homeomorphisms in Table 10 are obtained by using the following method: for $L = 8^2_{12}$ or 8^2_{10}, we have $S^2 \times S^1$ as the 0-surgery space of one component of L and we can move the other 0-framed component of 8^2_{12} to that of 8^2_{10} within $S^2 \times S^1$, so we have the homeomorphism, and we can apply it for the other cases.

$\chi(8^2_{12},0)\cong\chi(8^2_{10},0)$, $\chi(8^2_{13},0)\cong\chi(10^2_{154},0)$, $\chi(10^2_{163},0)\cong\chi(10^2_{170},0)$,
$\chi(5^2_1,0)\cong\chi(10^2_{174},0)\cong\chi(10^2_{173},0)$, $\chi(9^2_{27},0)\cong\chi(9^2_{15},0)$,
$\chi(10^2_{31},0)\cong\chi(10^2_{36},0)\cong\chi(10^2_{50},0)$, $\chi(9^2_{13},0)\cong\chi(10^2_{143},0)$,
$\chi(8^4_3,0)\cong\chi(10^2_{150},0)(\cong\chi(0^2_1,0))$, $\chi(9^2_{18},0)\cong\chi(10^2_{145},0)$, $\chi(9^2_{33},0)\cong\chi(9^2_{32},0)$,
$\chi(10^2_{83},0)\cong\chi(10^2_{104},0)$, $\chi(9^2_{31},0)\cong\chi(10^2_{A},0)$, $\chi(10^2_{107},0)\cong\chi(10^2_{106},0)$,
$\chi(10^2_{84},0)\cong\chi(10^2_{105},0)$, $\chi(10^2_{56},0)\cong\chi(10^2_{34},0)$, $\chi(10^2_{76},0)\cong\chi(10^2_{78},0)$

Table 10

So we omit
$8^2_{10},\ 8^4_1,\ 10^2_{154},\ 10^2_{170},\ 10^2_{174},\ 10^4_7,\ 9^2_{15},\ 10^2_{36},\ 10^2_{143},\ 10^2_{150},\ 10^2_{145},$
$10^2_{50},\ 9^2_{32},\ 10^2_{104},\ 10^2_{A},\ 10^2_{106},\ 10^2_{173},\ 10^2_{105},\ 10^4_9,\ 10^2_{34},\ 10^2_{78}$
from the sequence. For the rest of the links, we compute the two-variable Alexander polynomials, which are shown in Table 11.

L	$\Delta(\chi(L,0))$	L	$\Delta(\chi(L,0))$
5^2_1	1	9^2_{18}	$2-2t_1+2t_1^2$
7^2_4	$1+t_1^2$	10^2_{31}	$1-2t_1+3t_1^2-2t_1^3+t_1^4$
7^2_6	$1-t_1+t_1^2$	10^2_{107}	$1-3t_1+4t_1^2-3t_1^3+t_1^4$
7^2_3	2	10^2_{93}	$1-3t_1+4t_1^2-3t_1^3+t_1^4$
8^2_{12}	$1-2t_1+t_1^2$	9^2_{36}	$2-2t_1+2t_1^2$
8^2_{13}	$1-3t_1+t_1^2$	10^2_{178}	$1+2t_2+t_2^2$
8^4_3	0	10^2_{120}	$1-2t_1-2t_2+t_1^2+3t_1t_2+t_2^2-2t_1^2t_2-2t_1t_2^2+t_1^2t_2^2$
9^2_{13}	$1+t_1^2+t_1^4$	9^2_{33}	$2-3t_1+2t_1^2$
9^2_{55}	$1-t_1+t_1^2$	10^2_{81}	$1-3t_1+3t_1^2-3t_1^3+t_1^4$
9^2_{31}	$1-t_1+t_1^2-t_1^3+t_1^4$	10^2_{91}	$1-3t_1+4t_1^2-3t_1^3+t_1^4$
9^2_5	$t_1^2+2t_1t_2+t_2^2$	10^2_{26}	$1-t_1-t_2+2t_1t_2-t_1^2t_2-t_1t_2^2+t_1^2t_2^2$
9^2_{37}	$1-t_1+2t_1^2-t_1^3+t_1^4$	10^2_{79}	$1-2t_1+4t_1^2-2t_1^3+t_1^4$
9^2_{41}	$1-t_1+3t_1t_2-t_1t_2^2+t_1^2t_2^2$	10^2_{48}	$1-t_1-t_2+3t_1t_2-t_1^2t_2-t_1t_2^2+t_1^2t_2^2$
9^2_{25}	$1-4t_1+t_1^2$	10^2_{83}	$1-3t_2+3t_2^2-3t_2^3+t_2^4$
9^2_4	$t_1+t_2+t_1t_2+t_1^2t_2+t_1t_2^2$	10^2_{158}	3
9^2_{27}	$2-t_1+2t_1^2$	10^2_{116}	$1-t_1-2t_2+t_1^2+3t_1t_2+t_2^2-2t_1^2t_2-t_1t_2^2+t_1^2t_2^2$
10^2_{56}	$1-2t_1+t_1^2-2t_1^3+t_1^4$	10^2_{27}	$1-t_1-t_2+4t_1t_2-t_1^2t_2-t_1t_2^2+t_1^2t_2^2$
10^2_{169}	$1+t_1^4$	10^2_{84}	$1-3t_1+5t_1^2-3t_1^3+t_1^4$
10^2_{163}	$1-2t_1+t_1^2$	10^2_{66}	$1-t_1-t_2+3t_1t_2-t_1^2t_2-t_1t_2^2+t_1^2t_2^2$
10^2_{76}	$1-2t_1+2t_1^2-2t_1^3+t_1^4$	10^2_{148}	$t_1+t_2-2t_1t_2+t_1^2t_2+t_1t_2^2$
10^2_{162}	$1+t_1^2$	10^2_{152}	$t_1-t_2-t_1^2t_2+t_1t_2^2$
10^2_{94}	$1-3t_1+2t_1^2-3t_1^3+t_1^4$	10^2_{25}	$1-t_1-t_2-t_1^2t_2-t_1t_2^2+t_1^2t_2^2$
10^2_{176}	$1+2t_2+t_2^2$	10^2_{146}	$1+t_1^2-t_1t_2+t_2^2+t_1^2t_2^2$
9^2_9	$t_1+t_2-t_1t_2+t_1^2t_2+t_1t_2^2$	10^2_{103}	$1-t_1-t_2+t_1^2+t_1t_2+t_2^2-t_1^2t_2-t_1t_2^2+t_1^2t_2^2$

Table 11

Thus we conclude that the manifolds are different from each other except the cases in Table 12 by using Lemma 5.2 [14, 18], where we write $f(t_1, t_2) \equiv g(t_1, t_2)$ for $f(t_1, t_2),\ g(t_1, t_2) \in \mathbf{Z}[t_1^{\pm 1}, t_2^{\pm 1}]$ if there is an automorphism ψ of the free abelian group $(t_1, t_2 \,|\, t_1 t_2 = t_2 t_1)$ such that $f(t_1, t_2) = \pm t_1^{s_1} t_2^{s_2} g(\psi(t_1), \psi(t_2))$ for some integers s_1, s_2.

$\Delta(\chi(7_4^2, 0)) \equiv \Delta(\chi(10_{162}^2, 0))$, $\Delta(\chi(7_6^2, 0)) \equiv \Delta(\chi(9_{55}^2, 0))$,
$\Delta(\chi(8_{12}^2, 0)) \equiv \Delta(\chi(10_{163}^2, 0))$, $\Delta(\chi(9_5^2, 0)) \equiv \Delta(\chi(10_{176}^2, 0)) \equiv \Delta(\chi(10_{178}^2, 0))$,
$\Delta(\chi(9_9^2, 0)) \equiv \Delta(\chi(10_{146}^2, 0))$, $\Delta(\chi(9_{18}^2, 0)) \equiv \Delta(\chi(9_{36}^2, 0))$,
$\Delta(\chi(10_{81}^2, 0)) \equiv \Delta(\chi(10_{83}^2, 0))$, $\Delta(\chi(10_{107}^2, 0)) \equiv \Delta(\chi(10_{93}^2, 0)) \equiv \Delta(\chi(10_{91}^2, 0))$,
$\Delta(\chi(10_{48}^2, 0)) \equiv \Delta(\chi(10_{66}^2, 0))$.

Table 12

Lemma 5.2. Let M, M' be closed connected orientable 3-manifolds with $H_1(M) \cong H_1(M') \cong \mathbf{Z}^r$ and $\Delta_M(t_1, \ldots, t_r)$, $\Delta_{M'}(t_1, \ldots, t_r)$ their Alexander polynomials. If there is a homeomorphism $h : M \to M'$, then there is an automorphism ψ of the multiplicative free abelian group $(t_1, \ldots, t_r | t_i t_j = t_j t_i (i, j = 1, \ldots, r))$ with basis t_i $(i = 1, \ldots, r)$ such that

$$\Delta_M(t_1, \ldots, t_r) = \pm t_1^{s_1} \cdots t_r^{s_r} \Delta_{M'}(\psi(t_1), \ldots, \psi(t_r))$$

for some integers s_i $(i = 1, \ldots, r)$.

For the above 20 manifolds, we compute

$$\tau_5(\chi(L, 0)) = C(1 + (s^{-2} + 2 + s^2) \cdot f_L),$$

where $C = \frac{8}{5}\sin^2\frac{\pi}{5}$ and $s = e^{\frac{\pi i}{5}}$. We show these values of f_L in Table 13.

L	f_L	L	f_L
7_4^2	$-2s + 4s^2$	10_{162}^2	$3 - 3s + 3s^2 - 2s^3$
7_6^2	$6 - 4s + 2s^2 + s^3$	9_{55}^2	$2 - 6s + 5s^2 - 3s^3$
8_{12}^2	$4 - 5s + s^2 + 4s^3$	10_{163}^2	$4 - 4s^2 + 4s^3$
9_5^2	$6 - 4s + 2s^2 + s^3$	10_{176}^2	$1 + s - 3s^2 + s^3$
10_{178}^2	$1 + s + 2s^2 + s^3$	9_9^2	$5 - 7s + 4s^2$
10_{146}^2	$-2 + 2s - 2s^2 + 3s^3$	9_{18}^2	$5 - 12s + 9s^2 - 5s^3$
9_{36}^2	$3 - 8s + 3s^2 + 3s^3$	10_{81}^2	$2 - 11s + 15s^2 - 8s^3$
10_{83}^2	$-3 + 14s - 15s^2 + 7s^3$	10_{107}^2	$14 - 15s + 6s^2 + 4s^3$
10_{93}^2	$14 - 10s + s^2 + 4s^3$	10_{91}^2	$4 - 15s + 16s^2 - 11s^3$
10_{48}^2	$-4 + 11s - 13s^2 + 6s^3$	10_{66}^2	$7 - 6s + 7s^3$

Table 13

Since these values are not equivalent to each other, we have the enumeration of 3-manifolds in Case 3. We note here that some double covering spaces of $\chi(7^2_6, 0)$ and $\chi(9^2_{55}, 0)$ associated with the monodromy homomorphisms sending the meridians of trivial components of 7^2_6 and 9^2_{55} to $1 \in \mathbf{Z}_2$ and the meridians of the other components of 7^2_6 and 9^2_{55} to $0 \in \mathbf{Z}_2$ are homeomorphic.

Case 4. $H_1(\chi(L, 0)) \cong \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z}$.

The links with this condition in our group table are the following:

$$6^3_2 < 9^3_{12} < 9^3_{21} < 9^3_9 < 10^3_{33} < 10^3_{24}.$$

Their three-variable Alexander polynomials are in Table 14 and we see that the manifolds are different from each other by using Lemma 5.2.

$$\begin{array}{ll} \Delta(\chi(6^3_2, 0)) = 1, & \Delta(\chi(9^3_{12}, 0)) = 1 - 2t_1 + t_1^2, \\ \Delta(\chi(9^3_{21}, 0)) = 0, & \Delta(\chi(9^3_9, 0)) = 1 - t_1 + t_1^2, \\ \Delta(\chi(10^3_{33}, 0)) = 1 + 2t_1t_2 + t_1^2t_2^2, & \Delta(\chi(10^3_{24}, 0)) = 1 - 3t_1 + t_1^2. \end{array}$$

Table 14

Case 5. $H_1(\chi(L, 0)) \cong \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z}$

We have just one link 10^4_{21} with this condition in our group table, and $\chi(10^4_{21}, 0)$ appears in our manifold table.

Case 6. $H_1(\chi(L, 0)) \cong \mathbf{Z} \oplus \mathbf{Z}_2$.

The links with this condition in our group table are the following:

$$10^4_{15} < 10^4_8 < 10^4_{20} < 10^4_5.$$

For each manifold $M = \chi(L, 0)$, there is the maximal free abelian covering space $(\widetilde{M}, p)$, and we compute the first characteristic polynomial $\Delta(M)$ of the $\mathbf{Z}[t, t^{-1}]$-module $H_1(\widetilde{M}, p^{-1}(x))$, where $x \in M$:

$$\begin{array}{ll} \Delta(\chi(10^4_{15}, 0)) = 1 + t^2, & \Delta(\chi(10^4_8, 0)) = 1 - 4t + t^2, \\ \Delta(\chi(10^4_{20}, 0)) = 2, & \Delta(\chi(10^4_5, 0)) = 2 - 2t + 2t^2. \end{array}$$

We conclude that the manifolds are mutually distinct.

Case 7. $H_1(\chi(L, 0)) \cong \mathbf{Z} \oplus \mathbf{Z}_2 \oplus \mathbf{Z}_2$.

The links with this condition in our group table are the following:

$$8^3_9 < 8^3_{10} < 8^3_4 < 9^3_8 < 9^3_{20} < 10^3_{27}.$$

The homeomorphism $\chi(8^3_4, 0) \cong \chi(9^3_{20}, 0)$ is obtained by using a handle slide and remove $\chi(9^3_{20}, 0)$ from the sequence. For each link L except 9^3_{20}, let $(\widetilde{M}, p)$ be the maximal free abelian covering space of $M = \chi(L, 0)$. We compute the first characteristic polynomial $\Delta(M)$ and the second elementary ideal $E_2(M)$ of the $\mathbf{Z}[t, t^{-1}]$-module $H_1(\widetilde{M}, p^{-1}(x))$, where $x \in M$. We show them in Table 15 and we distinguish the manifolds.

L	$\Delta(M)$	$E_2(M)$	L	$\Delta(M)$	$E_2(M)$
8^3_9	$(1+t)^2$	$\langle 1+t \rangle$	8^3_{10}	4	$\langle 2 \rangle$
8^3_4	4	$\langle 2,\ 1+t^2 \rangle$	9^3_8	$(1+t)^2$	$\langle 4,\ 1+t \rangle$
10^3_{27}	$(3-t)(1-3t)$	$\langle 3-t,\ 1-3t \rangle$			

Table 15

Case 8. $H_1(\chi(L, 0)) \cong \mathbf{Z} \oplus \mathbf{Z}_3 \oplus \mathbf{Z}_3$

There is just one link 10^3_{56} with this condition in our group table and $\chi(10^3_{56}, 0)$ appears in our manifold table.

Case 9. $H_1(\chi(L, 0)) \cong \mathbf{Z}_2$

There are 12 links with this condition in our group table:

$6^3_3 < 6^3_1 < 7^3_1 < 8^3_3 < 9^3_{16} < 9^3_{15} < 9^3_4 < 10^3_{35} < 9^3_7 < 10^3_{53} < 10^3_{10} < 10^3_{54}$.

We see that $\chi(8^3_3, 0) \cong \chi(9^3_{16}, 0)$ and 9^3_{16} is omitted from the sequence. For the rest of the manifolds $\chi(L, 0)$, we compute the first homology groups of the double covering spaces $\chi(L, 0)_2$, which are shown in Table 16,

L	$H_1(\chi(L,0)_2)$	L	$H_1(\chi(L,0)_2)$	L	$H_1(\chi(L,0)_2)$	L	$H_1(\chi(L,0)_2)$
6^3_3	0	6^3_1	$\mathbf{Z}_3$	7^3_1	$\mathbf{Z}_5$	8^3_3	$\mathbf{Z}_7$
9^3_{15}	$\mathbf{Z}_5$	9^3_4	$\mathbf{Z}_{13}$	10^3_{35}	$\mathbf{Z}_{15}$	9^3_7	$\mathbf{Z}_9$
10^3_{53}	$\mathbf{Z}_9$	10^3_{10}	$\mathbf{Z}_{19}$	10^3_{54}	$\mathbf{Z}_{11}$		

Table 16

and conclude that the manifolds are different from each other except the cases $H_1(\chi(7^3_1, 0)_2) \cong H_1(\chi(9^3_{15}, 0)_2) \cong \mathbf{Z}_5$, $H_1(\chi(9^3_7, 0)_2) \cong H_1(\chi(10^3_{53}, 0)_2) \cong \mathbf{Z}_9$. For the first case we compute the first homology groups of the 5 fold covering spaces $(\chi(L, 0)_2)_5$ of $\chi(L, 0)_2$ and for the second case compute that of the 3 fold covering spaces $(\chi(L, 0)_2)_3$ of $\chi(L, 0)_2$ and have

$H_1((\chi(7^3_1, 0)_2)_5) \cong \mathbf{Z}_2 \oplus \mathbf{Z}_2 \oplus \mathbf{Z}_2 \oplus \mathbf{Z}_2$, $H_1((\chi(9^3_{15}, 0)_2)_5) \cong \mathbf{Z}_4 \oplus \mathbf{Z}_4 \oplus \mathbf{Z}_4 \oplus \mathbf{Z}_4$,
$H_1((\chi(9^3_7, 0)_2)_3) \cong \mathbf{Z}_2 \oplus \mathbf{Z}_6$, $H_1((\chi(10^3_{53}, 0)_2)_3) \cong \mathbf{Z}_4 \oplus \mathbf{Z}_{12}$,

so we conclude that the manifolds are different from each other.

Case 10. $H_1(\chi(L,0)) \cong \mathbf{Z}_4$.

We have 7 links with this condition in our group table:

$$8^3_1 < 8^3_2 < 9^3_6 < 9^3_{11} < 9^3_5 < 10^3_5 < 10^3_6.$$

By computing the first homology groups of the double covering spaces $\chi(L,0)_2$ of the manifolds $\chi(L,0)$, we see that the manifolds are different from each other. We show the homology groups in Table 17.

$H_1(\chi(8^3_1,0)_2) \cong \mathbf{Z}_{10}$, $H_1(\chi(8^3_2,0)_2) \cong \mathbf{Z}_{14}$, $H_1(\chi(9^3_6,0)_2) \cong \mathbf{Z}_{22}$,
$H_1(\chi(9^3_{11},0)_2) \cong \mathbf{Z}_{30}$, $H_1(\chi(9^3_5,0)_2) \cong \mathbf{Z}_{18}$, $H_1(\chi(10^3_5,0)_2) \cong \mathbf{Z}_{34}$,
$H_1(\chi(10^3_6,0)_2) \cong \mathbf{Z}_{38}$

Table 17

Case 11. $H_1(\chi(L,0)) \cong \mathbf{Z}_6$

There are 5 links with this condition in our group table:

$$10^3_1 < 10^3_2 < 10^3_{58} < 10^3_{64} < 10^3_{21}.$$

By computing the first homology groups of the double covering spaces $\chi(L,0)_2$ of the manifolds $\chi(L,0)$, we see that the manifolds are different from each other. We show the homology groups in Table 18.

$H_1(\chi(10^3_1,0)_2) \cong \mathbf{Z}_{21}$, $H_1(\chi(10^3_2,0)_2) \cong \mathbf{Z}_{33}$, $H_1(\chi(10^3_{58},0)_2) \cong \mathbf{Z}_3 \oplus \mathbf{Z}_3 \oplus \mathbf{Z}_3$,
$H_1(\chi(10^3_{64},0)_2) \cong \mathbf{Z}_3 \oplus \mathbf{Z}_3$, $H_1(\chi(10^3_{21},0)_2) \cong \mathbf{Z}_3 \oplus \mathbf{Z}_{15}$

Table 18

Case 12. $H_1(\chi(L,0)) \cong \mathbf{Z}_8$

The links in our group table with this condition are the following:

$$10^3_{50} < 10^3_{51} < 10^3_7 < 10^3_{52} < 10^3_8 < 10^3_{73} < 10^3_{41}.$$

The 3-manifolds obtained by the 0-surgery of the above three component links are homeomorphic to ± 8-surgery of some knots and we have:

$\chi(10^3_{50},0) \cong \chi(K_1,8)$, $\chi(10^3_{51},0) \cong \chi(K_2,-8)$, $\chi(10^3_7,0) \cong \chi(K_3,-8)$,
$\chi(10^3_{52},0) \cong \chi(K_4,-8)$, $\chi(10^3_8,0) \cong \chi(K_5,-8)$, $\chi(10^3_{73},0) \cong \chi(K_6,-8)$,
$\chi(10^3_{41},0) \cong \chi(K_7,-8)$.

The pictures of $K_1, K_2, \ldots, K_7$ are given in Fig. 5 and we see that we can omit 10^3_{73} from the sequence.

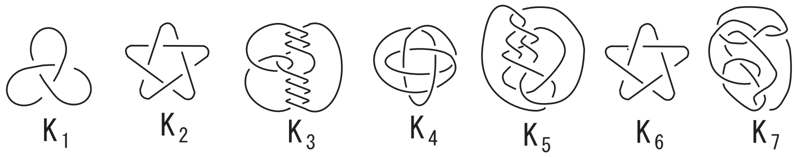

Fig. 5

For the rest of the manifolds $\chi(L,0)$, we compare the orders of the first homology groups of the n-fold covering spaces $\chi(L,0)_n$ of $\chi(L,0)$ for $n = 2,4,8$ and we distinguish the manifolds $\chi(L,0)$. We show the data in Table 19.

L	$\lvert H_1(\chi(L,0)_2)\rvert$	$\lvert H_1(\chi(L,0)_4)\rvert$	$\lvert H_1(\chi(L,0)_8)\rvert$
10^3_{50}	12	6	3
10^3_{51}	20	10	
10^3_{7}	12	6	147
10^3_{52}	12	54	
10^3_{8}	20	90	
10^3_{41}	52		

Table 19

Remark 5.3. Since we have $\chi(L,0) \cong \chi(K,\pm 8)$ for some knot K, we can compute $|H_1(\chi(L,0)_n)|$ in the following way. Let $\Sigma_n(K)$ be the n-fold covering space branched over K. Then we have $H_1(\chi(L,0)_n) \cong H_1(\chi(K,\pm 8)_n) \cong H_1(\Sigma_n(K)) \oplus \mathbf{Z}_{\frac{8}{n}}$ and we use Fox's formula $|H_1(\Sigma_n(K))| = |\prod_{k=1}^{n} \Delta(\omega^k)|$, where $\omega = e^{\frac{2\pi i}{n}}$ (See [8]).

Case 13. $H_1(\chi(L,0)) \cong \mathbf{Z}_2 \oplus \mathbf{Z}_2$

There are 60 links with this condition in our group table:

$4^2_1 < 6^2_3 < 7^2_5 < 8^2_9 < 8^2_{11} < 8^4_2 < 8^2_{14} < 9^2_1 < 9^2_2 < 9^2_{23} < 9^2_{57} < 9^2_{14} < 9^2_{59} < 9^2_{29} < 8^2_6 < 10^2_{141} < 9^2_7 < 10^2_{11} < 10^2_{155} < 9^2_{58} < 10^2_{110} < 9^2_6 < 10^2_{10} < 10^2_{18} < 9^2_{26} < 9^4_1 < 9^2_{38} < 10^2_{90} < 10^2_{35} < 9^2_{16} < 10^4_6 < 9^2_{60} < 9^2_{30} < 10^2_{74} < 10^2_{172} < 10^2_{156} < 10^2_{108} < 10^4_{19} < 10^2_{165} < 10^2_{72} < 9^2_{17} < 10^4_{16} < 10^4_{14} < 10^4_4 < 10^2_{181} < 10^2_{184} < 10^2_{19} < 9^2_{28} < 10^2_{37} < 10^2_{75} < 10^2_{30} < 10^2_{149} < 10^2_{144} < 10^2_{171} < 10^2_{151} < 10^2_{147} < 10^2_{54} < 10^2_{96} < 10^2_{55} < 10^2_{52}$.

The homeomorphisms $\chi(4^2_1,0) \cong \chi(8^4_2,0)$, $\chi(6^2_3,0) \cong \chi(9^4_1,0)$, $\chi(10^4_{19},0) \cong \chi(10^4_{14},0)$, $\chi(10^4_6,0) \cong \chi(10^4_4,0)$ are obtained by using handle slides.

The homeomorphisms

$\chi(4^2_1,0) \cong \chi(9^2_{60},0)$, $\chi(9^2_{26},0) \cong \chi(10^2_{172},0)$, $\chi(9^2_{16},0) \cong \chi(10^2_{156},0)$
$\chi(9^2_{59},0) \cong \chi(10^2_{165},0)$, $\chi(10^2_{72},0) \cong \chi(10^2_{37},0)$, $\chi(10^2_{35},0) \cong \chi(10^2_{75},0)$,
$\chi(10^2_{155},0) \cong \chi(10^2_{144},0)$, $\chi(9^2_{28},0) \cong \chi(10^2_{171},0)$

are obtained by setting one link $L = O \cup K$ and the other $L' = O \cup K'$ and deforming 0-framed K into 0-framed K' in $\chi(O, 0) = S^2 \times S^1$ as in Case 3, where O is a trivial component of L and L'. So we omit

$8^4_2,\ 9^4_1,\ 9^2_{60},\ 10^2_{172},\ 10^2_{156},\ 10^2_{165},\ 10^4_{14},\ 10^4_4,\ 10^2_{37},\ 10^2_{75},\ 10^2_{144},\ 10^2_{171}$

from the sequence. For the rest of the manifolds we compute the first homology groups of the three double covering spaces $M_{t_1t_2}(L),\ M_{t_1}(L),\ M_{t_2}(L)$, each of which is the covering space of $\chi(L, 0)$ corresponding to the epimorphism $\nu_{12},\ \nu_1,\ \nu_2 : H_1(\chi(L, 0)) = (t_1 \,|\, t_1^2 = 1) \oplus (t_2 \,|\, t_2^2 = 1) \to (t \,|\, t^2 = 1)$ respectively, where $\nu_{12}(t_1) = \nu_{12}(t_2) = t,\ \nu_1(t_1) = t,\ \nu_1(t_2) = 1,\ \nu_2(t_1) = 1,\ \nu_2(t_2) = t$. We show the groups in Table 20. Here we write $n = \mathbf{Z}_n$ and $m, n = \mathbf{Z}_m \oplus \mathbf{Z}_n$ for integers m, n and merely write $M_{t_1t_2} = M_{t_1t_2}(L),\ M_{t_1} = M_{t_1}(L),\ M_{t_2} = M_{t_2}(L)$. For each L, we denote by $H(L)$ the unordered triplet $H_1(M_{t_1t_2}(L)),\ H_1(M_{t_1}(L)),\ H_1(M_{t_2}(L))$. Then we see that the manifolds are different from each other except the following cases:

$$H(8^2_9) = H(10^2_{151}),\quad H(9^2_1) = H(10^2_{149}),\qquad H(9^2_{57}) = H(10^2_{141}),$$

L	$H_1(M_{t_1t_2})$	$H_1(M_{t_1})$	$H_1(M_{t_2})$	L	$H_1(M_{t_1t_2})$	$H_1(M_{t_1})$	$H_1(M_{t_2})$
4^2_1	4	4	4	6^2_3	12	12	12
7^2_5	20	4	20	8^2_9	28	12	28
8^2_{11}	28	20	28	8^2_{14}	$3, 12$	$3, 12$	20
9^2_1	20	12	12	9^2_2	28	4	4
9^2_{23}	$3, 12$	4	4	9^2_{57}	20	12	20
9^2_{14}	36	36	12	9^2_{59}	4	4	12
9^2_{29}	44	44	4	8^2_6	20	20	20
10^2_{141}	12	20	20	9^2_7	44	12	12
10^2_{11}	60	4	4	10^2_{155}	36	4	36
9^2_{58}	28	28	4	10^2_{110}	84	20	12
9^2_6	36	4	4	10^2_{10}	52	12	12
10^2_{18}	68	4	4	9^2_{26}	52	4	52
9^2_{38}	60	60	4	10^2_{90}	84	84	28
10^2_{35}	76	4	76	9^2_{16}	44	12	44
10^4_6	$2, 10$	$\mathbf{Z}$	$2, 10$	9^2_{30}	52	12	52
10^2_{74}	68	12	68	10^2_{108}	92	28	4
10^4_{19}	$2, 6$	8	$2, 6$	10^2_{72}	84	4	84
9^2_{17}	36	4	36	10^4_{16}	$2, 2$	$\mathbf{Z}$	$2, 2$
10^2_{181}	36	28	4	10^2_{184}	60	4	4
10^2_{19}	76	12	12	9^2_{28}	44	44	4
10^2_{30}	60	60	20	10^2_{149}	12	12	20
10^2_{151}	12	28	28	10^2_{147}	36	20	44
10^2_{54}	84	4	4	10^2_{96}	$5, 20$	$5, 20$	12
10^2_{55}	44	28	44	10^2_{52}	60	52	52

Table 20

For the above 12 links, we compute

$$\tau_5(\chi(L,0)) = C(1 + (s^{-2} + 2 + s^2) \cdot f_L),$$

where $C = \frac{8}{5}\sin^2\frac{\pi}{5}$ and $s = e^{\frac{\pi i}{5}}$. We show these values of f_L in Table 21.

L	f_L	L	f_L
8^2_9	$5 - 6s + s^2 + s^3$	10^2_{151}	$2 - 5s + 2s^2 + 3s^3$
9^2_1	$2 + 2s^2 - 2s^3$	10^2_{149}	$2 - 5s + 2s^2 - 2s^3$
9^2_{57}	$4s - 4s^2 + s^3$	10^2_{141}	$-1 + s - 2s^2$
9^2_{29}	$3 - 7s + 10s^2 - 6s^3$	9^2_{28}	$-2 + 8s - 10s^2 + 4s^3$
10^2_{11}	$6 - 8s + 4s^2 + 2s^3$	10^2_{184}	$6 - 3s - 6s^2 + 7s^3$
10^2_{155}	$1 + 7s - 6s^2 + 2s^3$	9^2_{17}	$1 + 2s - 6s^2 + 7s^3$

Table 21

Since these values of f_L are not equivalent to each other, we have the enumeration of 3-manifolds in Case 13.

Case 14. $H_1(\chi(L,0)) \cong \mathbf{Z}_3 \oplus \mathbf{Z}_3$

There are 21 links with this condition in our group table:

$6^2_1 < 6^2_2 < 8^2_5 < 9^2_{51} < 8^2_3 < 9^2_{20} < 9^2_{40} < 10^2_{133} < 10^2_{38} < 10^2_{135} < 10^2_{63} < 10^4_{11} < 10^4_{10} < 9^2_{22} < 10^2_{179} < 10^2_{175} < 10^2_{142} < 10^2_{97} < 9^2_{24} < 10^2_{100} < 10^2_{23}$.

The homeomorphisms $\chi(6^2_2,0) \cong \chi(10^4_{11},0)$, $\chi(6^2_1,0) \cong \chi(10^4_{10},0)$ are obtained by using handle slides. The homeomorphisms

$$\chi(6^2_1,0) \cong \chi(9^2_{51},0), \quad \chi(8^2_5,0) \cong \chi(10^2_{135},0),$$
$$\chi(6^2_2,0) \cong \chi(10^2_{179},0), \quad \chi(8^2_3,0) \cong \chi(10^2_{175},0)$$

are obtained by setting one link $L = O \cup K$ and the other $L' = O \cup K'$ and deforming 0-framed K into 0-framed K' in $\chi(O,0) = S^2 \times S^1$ as in Case 3 and Case 13. So we omit

$$9^2_{51},\ 10^2_{135},\ 10^4_{11},\ 10^4_{10},\ 10^2_{179},\ 10^2_{175}$$

from the sequence. For the rest of the links, we compute the first homology groups of the four triple covering spaces $M_{t_1t_2}(L)$, $M'_{t_1t_2}(L)$, $M_{t_1}(L)$, $M_{t_2}(L)$, each of which is the covering space of $\chi(L,0)$ corresponding to the epimorphism ν_{12}, ν'_{12}, ν_1, $\nu_2 : H_1(\chi(L,0)) = (t_1 \,|\, t_1^3 = 1) \oplus (t_2 \,|\, t_2^3 = 1) \to (t \,|\, t^3 = 1)$ respectively, where $\nu_{12}(t_1) = \nu_{12}(t_2) = t$, $\nu'_{12}(t_1) = t$, $\nu'_{12}(t_2) = t^{-1}$, $\nu_1(t_1) = t$, $\nu_1(t_2) = 1$, $\nu_2(t_1) = 1$, $\nu_2(t_2) = t$. The sets of four groups distinguish the manifolds completely and we finish the classification in Case 14. We show the sets of four groups in Table 22. Here we write $n_1, \ldots, n_k = \mathbf{Z}_{n_1} \oplus \cdots \oplus \mathbf{Z}_{n_k}$ for positive integers $n_1, \ldots, n_k$ and merely write $M_{t_1t_2} = M_{t_1t_2}(L)$, $M'_{t_1t_2} = M'_{t_1t_2}(L)$, $M_{t_1} = M_{t_1}(L)$, $M_{t_2} = M_{t_2}(L)$.

L	$H_1(M_{t_1t_2})$	$H_1(M'_{t_1t_2})$	$H_1(M_{t_1})$	$H_1(M_{t_2})$
6^2_1	$\mathbf{Z}\oplus\mathbf{Z}$	$3,9$	$3,9$	$3,9$
6^2_2	$3,3,3$	$3,3,3$	$\mathbf{Z}\oplus\mathbf{Z}$	$\mathbf{Z}\oplus\mathbf{Z}$
8^2_5	$6,18$	$9,27$	$3,9$	$3,9$
8^2_3	$3,9$	$9,27$	$6,18$	$6,18$
9^2_{20}	$3,3,3$	$3,12,12$	$\mathbf{Z}\oplus\mathbf{Z}$	$9,27$
9^2_{40}	$6,18$	$18,54$	$3,9$	$2,2,6,18$
10^2_{133}	$3,3,3$	$3,6,6$	$\mathbf{Z}\oplus\mathbf{Z}$	$9,27$
10^2_{38}	$3,9$	$18,54$	$6,18$	$15,45$
10^2_{63}	$9,27$	$12,36$	$6,18$	$3,9$
9^2_{22}	$9,27$	$12,36$	$3,9$	$3,9$
10^2_{142}	$2,6,6,6$	$3,12,12$	$\mathbf{Z}\oplus\mathbf{Z}$	$\mathbf{Z}\oplus\mathbf{Z}$
10^2_{97}	$18,54$	$21,63$	$6,18$	$3,9$
9^2_{24}	$2,2,6,18$	$18,54$	$6,18$	$6,18$
10^2_{100}	$9,27$	$24,72$	$15,45$	$6,18$
10^2_{23}	$3,24,24$	$3,15,15$	$9,27$	$9,27$

Table 22

Case 15. $H_1(\chi(L,0)) \cong \mathbf{Z}_4 \oplus \mathbf{Z}_4$.

$$8^2_1 < 8^2_2 < 9^2_{53} < 8^2_4 < 9^2_{61} < 10^2_6 < 10^2_{46} < 10^2_{177} < 10^2_{17}$$

hold this condition and we see that the manifolds $\chi(L,0)$ are mutually distinct by computing the first homology groups of the three double covering spaces $M_{t_1t_2}(L)$, $M_{t_1}(L)$, $M_{t_2}(L)$, each of which is the covering space of $\chi(L,0)$ corresponding to the epimorphism $\nu_{12},\ \nu_1,\ \nu_2 : H_1(\chi(L,0)) = (t_1\,|\,t_1^4=1)\oplus(t_2\,|\,t_2^4=1) \to (t\,|\,t^2=1)$ respectively, where $\nu_{12}(t_1)=\nu_{12}(t_2)=t,\ \nu_1(t_1)=t,\ \nu_1(t_2)=1,\ \nu_2(t_1)=1,\ \nu_2(t_2)=t$. We show the data in Table 23. Here we write $l,m,n = \mathbf{Z}_l\oplus\mathbf{Z}_m\oplus\mathbf{Z}_n$ for integers l,m,n, especially $0,m,n = \mathbf{Z}\oplus\mathbf{Z}_m\oplus\mathbf{Z}_n$ and write $M_{t_1t_2}=M_{t_1t_2}(L),\ M_{t_1}=M_{t_1}(L),\ M_{t_2}=M_{t_2}(L)$.

L	$H_1(M_{t_1t_2})$	$H_1(M_{t_1})$	$H_1(M_{t_2})$	L	$H_1(M_{t_1t_2})$	$H_1(M_{t_1})$	$H_1(M_{t_2})$
8^2_1	$2,2,8$	$2,2,8$	$2,2,8$	8^2_2	$2,2,16$	$0,2,2$	$0,2,2$
9^2_{53}	$0,2,6$	$2,2,16$	$2,2,16$	8^2_4	$2,2,24$	$2,2,8$	$2,2,8$
9^2_{61}	$0,2,10$	$2,2,16$	$2,2,16$	10^2_6	$2,2,40$	$2,2,8$	$2,2,8$
10^2_{46}	$2,2,48$	$0,2,2$	$0,2,2$	10^2_{177}	$2,2,40$	$2,2,24$	$2,2,8$
10^2_{17}	$2,2,56$	$2,2,8$	$2,2,8$				

Table 23

Case 16. $H_1(\chi(L,0)) \cong \mathbf{Z}_5 \oplus \mathbf{Z}_5$.

There is just one link 10^2_1 with this condition in our group table, and $\chi(10^2_1,0)$ appears in our manifold table.

6. The table of 3-manifolds with lengths up to 10

We unify the cases 1 to 16 to obtain a table of the closed connected orientable 3-manifolds with lengths up to 10. In the table, $\mathbf{x}$-line denotes the lattice point $\mathbf{x}$ with length up to 10 satisfying the condition that $cl\beta(\mathbf{x})$ is a prime link and $\sigma(cl\beta(\mathbf{x})) = \mathbf{x}$, the L-line denotes the link $cl\beta(\mathbf{x})$ by using the order of the links Conway enumerated at the end of [5]. The π-line denotes the enumeration of the fundamental groups, where the groups $\pi_1(E(L))$ with the mark $\times$ should be deleted. The M-line denotes the enumeration order of the manifolds $\chi(L, 0)$ given by the links L with $\sigma(L) = \mathbf{x}$.

In Appendix, we show pictures of two or more component links with 10 crossings in [5] together with 7 omissions. We use the notation 10^{α}_{β}, where α denotes the number of components and β denotes the order Conway enumerated in [5]. For the links 10^{α}_{β} included in [5], C means the Conway's code and for the 7 omissions 10^{α}_{β} ($\beta = A, B, C, D$), x means the lattice point $\sigma(10^{\alpha}_{\beta})$.

$\mathbf{x}$	L	π	M	T	D
0	O		1	Seifert fibred	$S^1 \times S^2$
1^2	2_1^2		2	Seifert fibred	S^3
1^3	3_1		3	Torus bundle	$T \times I/[1,1\|-1,0]$
1^4	4_1^2		4	Seifert fibred	$SFS[S^2:(2,1)(2,1)(2,-1)]$
$1,-2,1,-2$	4_1		5	Torus bundle	$T \times I/[2,1\|1,1]$
1^5	5_1		6	Seifert fibred	$SFS[S^2:(2,1)(5,2)(10,-9)]$
$1^2,-2,1,-2$	5_1^2		7	Seifert fibred	$SFS[T:(1,1)]$
1^6	6_1^2		8	Seifert fibred	$SFS[S^2:(3,1)(3,1)(3,-1)]$
$1^3,2,-1,2$	5_2		9	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[0,1\|1,-1]$
$1^3,-2,1,-2$	6_2		10	Hyperbolic	Hyperbolic : 3.77082945111
$1^2,2,1^2,2$	6_3^3		11	Lens	RP^3
$1^2,-2,1^2,-2$	6_1^3		12	Seifert fibred	$SFS[S^2:(2,1)(3,1)(4,-3)]$
$1^2,-2,1,-2^2$	6_3		13	Hyperbolic	Hyperbolic : 4.05976642564
$1,-2,1,-2,1,-2$	6_2^3		14	Torus bundle	$T \times S^1$
$1,-2,1,3,-2,3$	6_3^2		15	Seifert fibred	$SFS[S^2:(2,1)(2,1)(2,1)(3,-5)]$
1^7	7_1		16	Seifert fibred	$SFS[S^2:(2,1)(7,3)(14,-13)]$
$1^4,2,-1,2$	6_2^2		17	Non-prime	$L(3,1)\sharp L(3,1)$
$1^4,-2,1,-2$	7_1^2		18	Hyperbolic	$Hyp2.25976713(0)$
$1^3,2,1^2,2$	7_7^2	×	×	Seifert fibred	$SFS[S^2:(2,1)(2,1)(6,-5)]$
$1^3,2,-1^2,2$	7_8^2	×	×	Seifert fibred	$SFS[T:(1,1)]$
$1^3,-2,1^2,-2$	7_4^2		19	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[-1,8\|0,1]$
$1^3,-2,1,-2^2$	7_2^2		20	Seifert fibred	$SFS[S^2:(2,1)(5,1)(7,-5)]$
$1^2,-2,1^2,-2^2$	7_5^2		21	Non-geometric($1Hyp,1SFS$)	$SFS[D:(2,1)(2,-1)]\cup Hyp2.02988321(L104001)$
$1^2,-2,1,-2,1,-2$	7_6^2		22	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,-2)]\cup Graph[S^2+3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2,2,-1,-3,2,-3$	6_1		23	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[0,1\|1,-2]$
$1^2,-2,1,3,-2,3$	7_6		24	Hyperbolic	Hyperbolic : 6.18027441937
$1,-2,1,-2,3,-2,3$	7_7		25	Hyperbolic	Hyperbolic : 6.3326666425
$1,-2,1,3,-2^2,3$	7_1^3		26	Seifert fibred	$SFS[S^2:(2,1)(4,1)(5,-4)]$
1^8	8_1^2		27	Seifert fibred△	$SFS[S^2:(4,1)(4,1)(4,-1)]$
$1^5,2,-1,2$	7_3		28	Hyperbolic	Hyperbolic : 4.21823364488
$1^5,-2,1,-2$	8_2		29	Hyperbolic	Hyperbolic : 4.70364205913
$1^4,2,1^2,2$	8_7^3	×	×	Lens	$L(4,1)$
$1^4,2,-1^2,2$	8_8^3	×	×	Seifert fibred	$SFS[S^2:(2,1)(2,1)(3,-2)]$
$1^4,-2,1^2,-2$	8_1^3		30	Seifert fibred	$SFS[S^2:(2,1)(5,2)(6,-5)]$
$1^3,2,1^3,2$	8_{19}		31	Seifert fibred	$SFS[S^2:(3,2)(4,1)(12,-11)]$
$1^3,2,-1^3,2$	8_{20}		32	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)]\cup/mSFS[D:(3,1)(3,2)]$, $m=[0,1\|1,0]$
$1^3,-2,1^3,-2$	8_5		33	Hyperbolic	Hyperbolic : 6.73630906712
$1^4,2,-1,2^2$	7_5		34	Hyperbolic	Hyperbolic : 5.98781044336
$1^4,-2,1,-2^2$	8_7		35	Hyperbolic	Hyperbolic : 6.11165991536
$1^3,2,-1^2,2^2$	8_{21}		36	Hyperbolic	Hyperbolic : 5.3334895669
$1^3,-2,1^2,-2^2$	8_{10}		37	Hyperbolic	Hyperbolic : 7.7900159735
$1^3,2,-1,2,-1,2$	8_9^3		38	Seifert fibred	$KB/n^2\,\tilde{\times}\,S^1$
$1^3,-2,1,-2,1,-2$	8_5^3		39	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[-1,3\|1,-2]$
$1^2,-2,1^2,-2,1,-2$	8_{16}		40	Hyperbolic	Hyperbolic : 9.78375114087
$1^3,-2,1,-2^3$	8_9		41	Hyperbolic	Hyperbolic : 5.65624417666
$1^3,-2^2,1,-2^2$	8_2^3		42	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)]\cup/mSFS[D:(2,1)(3,2)]$, $m=[0,1\|1,0]$
$1^2,-2,1,-2,1,-2^2$	8_{17}		43	Hyperbolic	Hyperbolic : 9.65085003623
$1^2,-2,1,-2^2,1,-2$	8_6^3		44	Seifert fibred	$SFS[S^2:(2,1)(2,1)(3,1)(3,-4)]$
$1^2,2^2,1^2,2^2$	8_{10}^3		45	Non-prime	$SFS[S^2:(2,1)(2,1)(2,-1)]\sharp S^2\times S^1$
$1^2,-2^2,1^2,-2^2$	8_4^3		46	Graph manifold($1SFS$)	Non-or, $g=2+2\text{punctures}/n^2\,\tilde{\times}\,S^1/[0,1\|1,0]$
$1,-2,1,-2,1,-2,1,-2$	8_{18}		47	Hyperbolic	Hyperbolic : 11.1472182257
$1^3,2,-1,-3,2,-3$	7_3^2		48	Seifert fibred	$SFS[T:(2,1)]$
$1^3,-2,1,3,-2,3$	8_5^2		49	Hyperbolic	Hyperbolic : 2.54158501007
$1^2,2,1^2,-3,2,-3$	8_{16}^2	×	×	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)]\cup/mSFS[D:(2,1)(3,1)]$, $m=[-3,4\|-2,3]$
$1^2,2,-1^2,-3,2,-3$	8_{15}^2	×	×	Seifert fibred	$SFS[T:(1,1)]$
$1^2,-2,1^2,3,-2,3$	8_9^2		50	Non-geometric($1Hyp,1SFS$)	$SFS[D:(2,1)(2,-1)]\cup Hyp2.82812209(L105002)$
$1^2,-2,1,-2,3,-2,3$	8_8^2		51	Hyperbolic	Hyperbolic : 5.20080525939
$1^2,-2,1,3,-2^2,3$	8_{12}^2		52	Non-geometric($2Hyp$)	Two-cusped $Hyp3.66386238(L205001)$, cusps truncated and identified
$1,-2,1,-2,1,3,-2,3$	8_{13}^2		53	Non-geometric($1Hyp,1SFS$)	$Hyp2.02988321(L104001)\cup Graph[S^2+3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2,-2,1,3,-2,3^2$	8_7^2		54	Hyperbolic	Hyperbolic : 5.94064222038
$1,-2,1,-2,3,-2^2,3$	8_{10}^2		×	Non-geometric($2Hyp$)	Two-cusped $Hyp3.66386238(L205001)$, cusps truncated and identified
$1,-2,1,3,-2^3,3$	8_{11}^2		55	Non-geometric($1Hyp,1SFS$)	$SFS[D:(2,1)(2,-1)]\cup Hyp2.82812209(L105002)$
$1,2^2,1,3,2^2,3$	8_3^4		56	Non-prime	$S^2\times S^1\sharp S^2\times S^1$
$1,2^2,1,3,-2^2,3$	8_2^4		×	Seifert fibred	$SFS[S^2:(2,1)(2,1)(2,-1)]$
$1,-2^2,1,3,-2^2,3$	8_1^4		×	Seifert fibred	$SFS[T:(1,1)]$

$\mathbf{x}$	L	π	M	T	D
$1,-2,3,-2,1,-2,3,-2$	8^2_{14}		57	Graph manifold($3SFS$)	$SFS[D:(2,1)(3,1)]$ $\cup/m$Non-or, $g=1+2$punctures$/n^2 \times \tilde{S}^1$ $\cup/nSFS[D:(2,1)(3,1)]$, $m=[1,-1 \mid 0,1], n=[1,1 \mid 0,1]$
$1,-2,1,3,-2,-4,3,-4$	8_{12}		58	Hyperbolic	Hyperbolic : 7.64659248194
1^9	9_1		59	Seifert fibred	$SFS[S^2:(2,1)(9,4)(18,-17)]$
$1^6,2,-1,2$	8^2_2		60	Seifert fibred	$SFS[RP^2/n^2:(2,1)(2,3)]$
$1^6,-2,1,-2$	9^2_1		61	Hyperbolic	Hyperbolic : 3.64868864441
$1^5,2,1^2,2$	9^2_{43}	$\times$	$\times$	Seifert fibred	$S^3/Q40$
$1^5,2,-1^2,2$	9^2_{44}	$\times$	$\times$	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[-1,8 \mid 0,1]$
$1^5,-2,1^2,-2$	9^2_{13}		62	Graph manifold($1SFS$)	$SFS[A:(3,1)]/[-1,12 \mid 0,1]$
$1^4,2,1^3,2$	9^2_{49}	$\times$	$\times$	Seifert fibred	$SFS[S^2:(3,1)(3,2)(6,-5)]$
$1^4,2,-1^3,2$	9^2_{51}		$\times$	Seifert fibred	$SFS[S^2:(3,1)(3,1)(3,-1)]$
$1^4,-2,1^3,-2$	9^2_{19}		63	Hyperbolic	Hyperbolic : 5.7851808578
$1^4,-2,-1^3,-2$	9^2_{50}		64	Seifert fibred	$SFS[S^2:(2,1)(3,1)(13,-11)]$
$1^5,2,-1,2^2$	8^2_3		65	Hyperbolic	Hyperbolic : 3.3849452212
$1^5,-2,1,-2^2$	9^2_2		66	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(3,1)(5,3)]$, $m=[0,1 \mid 1,0]$
$1^4,2,-1^2,2^2$	9^2_{52}		67	Hyperbolic	Hyperbolic : 2.71245880839
$1^4,-2,1^2,-2^2$	9^2_{20}		68	Hyperbolic	Hyperbolic : 5.78356053
$1^4,2,-1,2,-1,2$	9^2_{55}		69	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,-1)] \cup Graph[S^2+3$punctures$]$ /[non-fibre-preserving gluing]
$1^4,-2,1,-2,1,-2$	9^2_{31}		70	Graph manifold($2SFS$)	$SFS[D:(2,1)(5,-3)] \cup Graph[S^2+3$punctures$]$ /[non-fibre-preserving gluing]
$1^3,2,1^3,2^2$	9^2_{53}		71	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(2,1)]$, $m=[-1,3 \mid 0,1]$
$1^3,2,-1^3,2^2$	9^2_{54}		72	Seifert fibred	$SFS[S^2:(3,1)(4,1)(5,-3)]$
$1^3,-2,1^3,2^2$	8^2_4		73	Hyperbolic	Hyperbolic : 3.66386237671
$1^3,-2,1^3,-2^2$	9^2_{23}		74	Hyperbolic	Hyperbolic : 5.56366782479
$1^3,2,-1^2,2,-1,2$	9^2_{57}		75	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(5,2)]$, $m=[-3,4 \mid -2,3]$
$1^3,-2,1^2,-2,1,-2$	9^2_{35}		76	Hyperbolic	Hyperbolic : 7.89790810521
$1^2,-2,1^2,-2,1^2,-2$	9^2_{40}		77	Hyperbolic	Hyperbolic : 8.62552153672
$1^4,-2,1,-2^3$	9^2_5		78	Graph manifold($1SFS$)	$SFS[A:(2,1)(2,1)]/[0,1 \mid 1,-1]$
$1^4,-2^2,1,-2^2$	9^2_{14}		79	Non-geometric($1Hyp,1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp3.16396323(L106001)$
$1^3,-2,1^2,-2^3$	9^2_{21}		80	Hyperbolic	Hyperbolic : 5.41903947919
$1^3,-2,1,-2,1,-2^2$	9^2_{34}		81	Hyperbolic	Hyperbolic : 7.26826102929
$1^3,-2,1,-2^2,1,-2$	9^2_{37}		82	Graph manifold($3SFS$)	$SFS[D:(2,1)(3,-2)] \cup Graph[S^2+3$punctures$]$ $\cup SFS[A:(2,1)]$
$1^3,2^2,1^2,2^2$	9^2_{59}		83	Seifert fibred	$SFS[S^2:(2,1)(2,1)(6,-5)]$
$1^3,-2^2,1^2,-2^2$	9^2_{29}		84	Non-geometric($1Hyp,1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp4.40083252(L106002)$
$1^2,-2,1^2,-2,1,-2^2$	9^2_{39}		85	Hyperbolic	Hyperbolic : 7.99382353937
$1^2,2,-1,2,1^2,2^2$	9^2_{61}		86	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(2,1)]$, $m=[-3,5 \mid -2,3]$
$1^2,-2,1,-2,1^2,-2^2$	9^2_{41}		87	Hyperbolic	Hyperbolic : 8.50516589391
$1^2,-2,1,-2,1,-2,1,-2$	9^2_{42}		88	Hyperbolic	Hyperbolic : 9.54670929
$1^4,2,-1,-3,2,-3$	8_6		89	Hyperbolic	Hyperbolic : 6.3326666425
$1^4,-2,1,3,-2,3$	9_{11}		90	Hyperbolic	Hyperbolic : 7.91268477624
$1^3,2,1^2,-3,2,-3$	9_{43}		91	Hyperbolic	Hyperbolic : 5.62696404695
$1^3,2,-1^2,-3,2,-3$	9_{44}		92	Hyperbolic	Hyperbolic : 5.3334895669
$1^3,-2,1^2,3,-2,3$	9_{36}		93	Hyperbolic	Hyperbolic : 9.47634776991
$1^3,-2,-1^2,3,-2,3$	9_{42}		94	Hyperbolic	Hyperbolic : 3.1772932786
$1^3,2,-1,2,3,-2,3$	7_2		95	Graph manifold($1SFS$)	$SFS[A:(3,2)]/[0,1 \mid 1,-1]$
$1^3,2,-1,2,-3,2,-3$	8_{14}		96	Hyperbolic	Hyperbolic : 8.19064265904
$1^3,-2,1,-2,3,-2,3$	9_{26}		97	Hyperbolic	Hyperbolic : 9.67554142603
$1^3,-2,1,-2,-3,2,-3$	8_4		98	Hyperbolic	Hyperbolic : 5.03627941776
$1^3,2,-1,-3,2^2,-3$	8^3_3		99	Seifert fibred	$SFS[S^2:(2,1)(4,1)(7,-5)]$
$1^3,-2,1,3,-2^2,3$	9^3_6		100	Hyperbolic	Hyperbolic : 2.82812208833
$1^2,2,1^2,2,-3,2,-3$	9^3_{13}	$\times$	$\times$	Seifert fibred	$SFS[S^2:(2,1)(3,1)(6,-5)]$
$1^2,2,-1^2,2,-3,2,-3$	9^3_{14}	$\times$	$\times$	Torus bundle	$T\times I/[2,1 \mid 1,1]$
$1^2,-2,1^2,-2,3,-2,3$	9^3_2		$\times$	Hyperbolic	Hyperbolic : 4.05976642564
$1^2,2,1^2,-3,2^2,-3$	9^3_{19}	$\times$	$\times$	Seifert fibred	$SFS[KB/n^2:(1,4)]$
$1^2,2,-1^2,-3,2^2,-3$	9^3_{18}	$\times$	$\times$	Torus bundle	$T\times S^1$
$1^2,-2,1^2,3,-2^2,3$	9^3_8		101	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,1)]$ $\cup/m$Non-or, $g=2+1$puncture$/n^2 \times \tilde{S}^1$, $m=[-1,1 \mid -1,2]$
$1^2,2,-1,2,1,3,-2,3$	9_{45}		102	Hyperbolic	Hyperbolic : 6.96575966545
$1^2,-2,1,-2,1,3,-2,3$	9_{32}		103	Hyperbolic	Hyperbolic : 12.217473
$1^2,-2,1,3,-2,1,3,-2$	9^3_{11}		104	Non-geometric($1Hyp,1SFS$)	$SFS[D:(2,1)(3,-2)]$ $\cup Hyp2.02988321(L104001)$
$1^3,2,-1,-3,2,-3^2$	8_8		105	Hyperbolic	Hyperbolic : 6.38487304951
$1^3,-2,1,3,-2,3^2$	9_{20}		106	Hyperbolic	Hyperbolic : 9.28781194114

$\mathbf{x}$	L	π	M	T	D
$1^2, -2, 1^2, 3, -2, 3^2$	9^3_1		$\times$	Hyperbolic	Hyperbolic : 3.77082945111
$1^2, 2, -1, 2^2, 3, -2, 3$	7_4		107	Graph manifold($2SFS$)	$SFS[A:(2,1)] \cup SFS[A:(2,1)]$
$1^2, 2, -1, 2^2, -3, 2, -3$	8_{11}		108	Hyperbolic	Hyperbolic : 7.18637867
$1^2, -2, 1, -2^2, 3, -2, 3$	9_{27}		109	Hyperbolic	Hyperbolic : 9.84166805475
$1^2, -2, 1, -2^2, -3, 2, -3$	8_{13}		110	Hyperbolic	Hyperbolic : 7.2598895782
$1^2, -2, 1, 3, 2^3, 3$	8_{15}		111	Hyperbolic	Hyperbolic : 9.3408131084
$1^2, -2, 1, 3, -2^3, 3$	9_{24}		112	Hyperbolic	Hyperbolic : 9.55789756113
$1^2, -2^2, 1, -2, 3, -2, 3$	9_{30}		113	Hyperbolic	Hyperbolic : 10.8265666034
$1^2, 2^2, 1, 3, 2^2, 3$	9^3_{17}	$\times$	$\times$	Lens	RP^3
$1^2, 2^2, 1, 3, -2^2, 3$	9^3_{16}		$\times$	Seifert fibred	$SFS[S^2:(2,1)(4,1)(7,-5)]$
$1^2, 2^2, 1, -3, 2^2, -3$	9^3_{15}		114	Seifert fibred	$SFS[S^2:(2,1)(5,2)(8,-7)]$
$1^2, -2^2, 1, 3, -2^2, 3$	9^3_4		115	Hyperbolic	Hyperbolic : 4.16399593361
$1, -2, 1, -2, 1, -2, 3, -2, 3$	9^3_{10}		116	Non-geometric($2Hyp$)	Two-cusped Hyp3.66386238(L205001), cusps truncated and identified
$1, -2, 1, -2, 1, 3, 2^2, 3$	9^3_{20}		$\times$	Graph manifold($1SFS$)	Non-or, $g = 2 + 2$punctures$/n^2 \tilde{\times} S^1/[0,1\|1,0]$
$1, -2, 1, -2, 1, 3, -2^2, 3$	9^3_{12}		117	Graph manifold($1SFS$)	$Graph[S^2 + 4$punctures$]$ /[two non-fibre-preserving gluings]
$1, -2, 1, -2, 1, -3, 2^2, -3$	9^3_{21}		118	Non-prime	$SFS[T:(1,1)] \sharp S^2 \times S^1$
$1, -2, 1, -2^2, 1, 3, -2, 3$	9_{33}		119	Hyperbolic	Hyperbolic : 12.1718408863
$1, 2, -1, 2, 3, -2, 1, -2, 3$	9_{46}		$\times$	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[-1,3\|1,-2]$
$1, -2, 1, -2, 3, -2, 1, -2, 3$	9_{34}		120	Hyperbolic	Hyperbolic : 13.2319091792
$1, -2, 1, -2, -3, -2, 1, -2, -3$	9_{47}		121	Hyperbolic	Hyperbolic : 9.12172161101
$1^2, -2, 1, -2, 3, -2, 3^2$	9_{31}		122	Hyperbolic	Hyperbolic : 10.8969637027
$1^2, -2, 1, 3, -2^2, 3^2$	9_{28}		123	Hyperbolic	Hyperbolic : 10.7591239048
$1, -2, 1, 3, -2, 1, 3, -2, 3$	9_{40}		124	Hyperbolic	Hyperbolic : 14.1762047553
$1^2, -2, 1, 3, -2, -4, 3, -4$	9^2_{11}		125	Hyperbolic	Hyperbolic : 7.45062795944
$1, -2, 1, -2^3, 3, -2, 3$	9_{17}		126	Hyperbolic	Hyperbolic : 8.82380876658
$1, -2, 1, -2, 3, -2^3, 3$	9_{22}		127	Hyperbolic	Hyperbolic : 9.91561518105
$1, -2, 1, 3, -2^4, 3$	9^3_5		128	Hyperbolic	Hyperbolic : 2.52741847732
$1, -2^2, 1, -2, 3, -2^2, 3$	9^3_9		129	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,1)]$ $\cup/m$Or, $g = 1 + 1$puncture $\times S^1$, $m = [-1,1\| -5,6]$
$1, -2^2, 3, -2, 1, -2, 3, -2$	9_{29}		130	Hyperbolic	Hyperbolic : 11.5051185131
$1, -2, 1, -2, 3, -2, -4, 3, -4$	9^2_{12}		131	Hyperbolic	Hyperbolic : 7.18740661552
$1, -2, 1, -2, -3, 2, 4, -3, 4$	8^2_6		132	Seifert fibred	$SFS[S^2:(2,1)(2,1)(2,1)(5,-8)]$
$1, -2, 1, 3, -2^2, -4, 3, -4$	9^2_{25}		133	Non-geometric($2Hyp$)	Two-cusped Hyp5.33348957 (L206003), cusps truncated and identified
1^{10}	10^2_1		134	Seifert fibred	$SFS[S^2:(5,1)(5,1)(5,-1)]$
$1^7, 2, -1, 2$	9_3		135	Hyperbolic	Hyperbolic : 4.83092217398
$1^7, -2, 1, -2$	10_2		136	Hyperbolic	Hyperbolic : 4.99437267858
$1^6, 2, 1^2, 2$	10^3_{44}	$\times$	$\times$	Lens	$L(6,1)$
$1^6, 2, -1^2, 2$	10^3_{45}	$\times$	$\times$	Seifert fibred	$SFS[S^2:(2,1)(4,1)(5,-3)]$
$1^6, -2, 1^2, -2$	10^3_1		137	Seifert fibred	$SFS[S^2:(2,1)(7,3)(8,-7)]$
$1^5, 2, 1^3, 2$	10_{124}		138	Seifert fibred	$SFS[S^2:(3,1)(5,3)(15,-14)]$
$1^5, 2, -1^3, 2$	10_{126}		139	Hyperbolic	Hyperbolic : 5.77744754562
$1^5, -2, 1^3, -2$	10_{46}		140	Hyperbolic	Hyperbolic : 7.58327695032
$1^5, -2, -1^3, -2$	10_{125}		141	Hyperbolic	Hyperbolic : 3.94662562617
$1^4, 2, 1^4, 2$	10^3_{50}		142	Seifert fibred	$SFS[S^2:(2,1)(2,1)(3,-1)]$
$1^4, 2, -1^4, 2$	10^3_{51}		143	Seifert fibred	$SFS[S^2:(2,1)(2,1)(5,-3)]$
$1^4, -2, 1^4, -2$	10^3_7		144	Hyperbolic	Hyperbolic : 4.15667542682
$1^6, 2, -1, 2^2$	9_6		145	Hyperbolic	Hyperbolic : 6.99234525345
$1^6, -2, 1, -2^2$	10_5		146	Hyperbolic	Hyperbolic : 6.98128746951
$1^5, 2, -1^2, 2^2$	10_{127}		147	Hyperbolic	Hyperbolic : 8.43820740649
$1^5, -2, 1^2, -2^2$	10_{47}		148	Hyperbolic	Hyperbolic : 8.99006008724
$1^5, 2, -1, 2, -1, 2$	10^3_{36}		149	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[1,-3\| -1,2]$
$1^5, -2, 1, -2, 1, -2$	10^3_{27}		150	Graph manifold($1SFS$)	$SFS[A:(3,2)]/[3,8\|2,5]$
$1^4, 2, 1^3, 2^2$	10_{139}		151	Hyperbolic	Hyperbolic : 4.72419307053
$1^4, 2, -1^3, 2^2$	10_{143}		152	Hyperbolic	Hyperbolic : 7.89586912976
$1^4, -2, 1^3, 2^2$	9_9		153	Hyperbolic	Hyperbolic : 7.82241098274
$1^4, -2, 1^3, -2^2$	10_{62}		154	Hyperbolic	Hyperbolic : 9.7685930006
$1^4, -2, -1^3, -2^2$	10_{141}		155	Hyperbolic	Hyperbolic : 5.70240292558
$1^4, 2, -1^2, 2, -1, 2$	10_{148}		156	Hyperbolic	Hyperbolic : 8.97340458053
$1^4, -2, 1^2, -2, 1, -2$	10_{85}		157	Hyperbolic	Hyperbolic : 11.4150123565
$1^3, 2, -1^3, 2, -1, 2$	10^3_A		$\times$	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)]$ $\cup/mSFS[D:(3,1)(3,2)]$, $m = [0,1\|1,0]$
$1^3, -2, 1^3, 2, -1, 2$	10^3_{52}		158	Seifert fibred	$SFS[S^2:(3,2)(4,1)(4,-3)]$
$1^3, -2, 1^3, -2, 1, -2$	10^3_{31}		159	Hyperbolic	Hyperbolic : 5.83941124979
$1^3, 2, -1^2, 2, -1^2, 2$	10_{155}		160	Hyperbolic	Hyperbolic : 7.40579464371
$1^3, -2, 1^2, -2, 1^2, -2$	10_{100}		161	Hyperbolic	Hyperbolic : 12.4436159172
$1^5, -2, 1, -2^3$	10_9		162	Hyperbolic	Hyperbolic : 7.21118612063
$1^5, -2^2, 1, -2^2$	10^3_2		163	Hyperbolic	Hyp2.19336432(Z_6)

$\mathbf{x}$	L	π	M	T	D
$1^4, -2, 1^2, -2^3$	10^3_8		164	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(3,1)(4,1)]$, $m = [1,1\|0,1]$
$1^4, 2, -1, 2, -1, 2^2$	10_{149}		165	Hyperbolic	Hyperbolic : 10.9330305113
$1^4, -2, 1, -2, 1, -2^2$	10_{82}		166	Hyperbolic	Hyperbolic : 11.495062
$1^4, 2, -1, 2^2, -1, 2$	10^3_{58}		167	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,1)] \cup /mSFS[D:(2,1)(3,1)]$, $m = [-5,6\|-4,5]$
$1^4, -2, 1, -2^2, 1, -2$	10^3_{35}		168	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,2)] \cup /mSFS[D:(2,1)(5,2)]$, $m = [-1,2\|0,1]$
$1^4, 2^2, 1^2, 2^2$	10^3_{62}		$\times$	Seifert fibred	$S^2 \times S^1$
$1^4, 2^2, -1^2, 2^2$	10^3_B		169	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(3,1)(3,2)]$, $m = [-1,2\|0,1]$
$1^4, -2^2, 1^2, -2^2$	10^3_{19}		170	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp3.66386238(m137)$
$1^3, -2, 1^3, 2^3$	9_{16}		171	Hyperbolic	Hyperbolic : 9.65931027945
$1^3, -2, 1^3, -2^3$	10_{64}		172	Hyperbolic	Hyperbolic : 9.9614305295
$1^3, 2, -1^2, 2, -1, 2^2$	10^3_{60}		$\times$	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[0,1\|1,-2]$
$1^3, -2, 1^2, -2, 1, -2^2$	10^3_{38}		173	Hyperbolic	Hyperbolic : 4.90949251141
$1^3, -2, 1^2, -2^2, 1, -2$	10_{94}		174	Hyperbolic	Hyperbolic : 12.4456792
$1^3, 2, -1, 2, 1^2, 2^2$	10_{161}		175	Hyperbolic	Hyperbolic : 5.38541418815
$1^3, 2, -1, 2, -1^2, 2^2$	10_{159}		176	Hyperbolic	Hyperbolic : 10.5092791382
$1^3, -2, 1, -2, 1^2, -2^2$	10_{106}		177	Hyperbolic	Hyperbolic : 13.112012888
$1^3, -2, 1, -2, 1, -2, 1, -2$	10_{112}		178	Hyperbolic	Hyperbolic : 13.9297487506
$1^3, 2^2, 1^3, 2^2$	10^3_{64}		179	Non-prime	$RP^3 \sharp L(3,1)$
$1^3, -2^2, 1^3, -2^2$	10^3_{21}		180	Hyperbolic	Hyperbolic : 4.30620760073
$1^2, 2, -1^2, 2, 1^2, 2^2$	10^3_{73}		$\times$	Seifert fibred	$SFS[S^2:(2,1)(2,1)(5,-3)]$
$1^2, -2, 1^2, -2, 1^2, -2^2$	10^3_{41}		181	Hyperbolic	Hyperbolic : 6.76702003092
$1^2, -2, 1^2, -2, 1, -2, 1, -2$	10_{116}		182	Hyperbolic	Hyperbolic : 14.6425908813
$1^2, -2, 1, -2, 1^2, -2, 1, -2$	10^3_{43}		183	Hyperbolic	Hyperbolic : 8.08797378863
$1^5, 2, -1, -3, 2, -3$	9^2_3		184	Hyperbolic	Hyperbolic : 3.90635745514
$1^5, -2, 1, 3, -2, 3$	10^2_6		185	Hyperbolic	Hyperbolic : 4.12490325181
$1^4, 2, 1^2, -3, 2, -3$	10^2_{133}		186	Hyperbolic	Hyperbolic : 3.99941348971
$1^4, 2, -1^2, -3, 2, -3$	10^2_{134}		$\times$	Seifert fibred	$SFS[S^2:(3,1)(4,1)(5,-3)]$
$1^4, -2, 1^2, 3, -2, 3$	10^2_{38}		187	Hyperbolic	Hyperbolic : 7.72255389698
$1^4, -2, -1^2, 3, -2, 3$	10^2_{132}	$\times$	$\times$	Hyperbolic	$Hyp1.39850888(0)$
$1^3, 2, 1^3, -3, 2, -3$	10^2_{141}		188	Hyperbolic	Hyperbolic : 4.72540158511
$1^3, 2, -1^3, -3, 2, -3$	10^2_{140}		$\times$	Seifert fibred	$SFS[S^2:(2,1)(3,1)(13,-11)]$
$1^3, -2, 1^3, 3, -2, 3$	10^2_{46}		189	Hyperbolic	Hyperbolic : 6.97947205053
$1^4, -2, 1, -2^4$	10_{17}		190	Hyperbolic	Hyperbolic : 6.32792646
$1^4, -2^2, 1, -2^3$	10_{48}		191	Hyperbolic	Hyperbolic : 8.64775778981
$1^3, -2, 1, -2, 1, -2^3$	10^3_{30}		192	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp2.02988321(L104001)$
$1^3, -2, 1, -2^2, 1, -2^2$	10_{91}		193	Hyperbolic	Hyperbolic : 12.1470795
$1^3, -2, 1, -2^3, 1, -2$	10^3_{33}		194	Seifert fibred	$SFS[T:(2,1)(2,-1)]$
$1^3, 2^2, 1^2, 2^3$	10_{152}		195	Hyperbolic	Hyperbolic : 8.31557310455
$1^3, -2^2, 1^2, -2^3$	10_{79}		196	Hyperbolic	Hyperbolic : 11.1659896902
$1^3, 2^2, -1, 2, -1, 2^2$	10_{157}		197	Hyperbolic	Hyperbolic : 12.141569098
$1^3, -2^2, 1, -2, 1, -2^2$	10_{104}		198	Hyperbolic	Hyperbolic : 12.8676817179
$1^2, -2, 1^2, -2^2, 1, -2^2$	10_{99}		199	Hyperbolic	Hyperbolic : 13.092890786
$1^2, -2, 1, -2, 1, -2, 1, -2^2$	10^3_{42}		200	Hyperbolic	Hyperbolic : 7.71574614581
$1^2, -2, 1, -2, 1, -2^2, 1, -2$	10_{118}		201	Hyperbolic	Hyperbolic : 14.4052313921
$1^2, -2, 1, -2^2, 1^2, -2^2$	10_{109}		202	Hyperbolic	Hyperbolic : 13.7446101017
$1, -2, 1, -2, 1, -2, 1, -2, 1, -2$	10_{123}		203	Hyperbolic	Hyperbolic : 16.0753782904
$1^4, 2, -1, 2, -3, 2, -3$	9^2_7		204	Hyperbolic	Hyperbolic : 6.3326666425
$1^4, -2, 1, -2, 3, -2, 3$	10^2_{11}		205	Hyperbolic	Hyperbolic : 6.34376088929
$1^4, -2, 1, -2, -3, 2, -3$	9^2_4		206	Hyperbolic	Hyperbolic : 4.71353681388
$1^4, 2, -1, -3, 2^2, -3$	9^2_{27}		207	Non-geometric($2Hyp$)	Two-cusped $Hyp4.05976643(L206002)$, cusps truncated and identified
$1^4, -2, 1, 3, -2^2, 3$	10^2_{56}		208	Non-geometric($2Hyp$)	Two-cusped $Hyp4.74949998(L207001)$, cusps truncated and identified
$1^3, 2, 1^2, 2, -3, 2, -3$	10^2_{136}		209	Hyperbolic	Hyperbolic : 3.16236728645
$1^3, 2, -1^2, 2, -3, 2, -3$	10^2_{139}		210	Hyperbolic	Hyperbolic : 6.22139631555
$1^3, -2, 1^2, -2, 3, -2, 3$	10^2_{44}		211	Hyperbolic	Hyperbolic : 8.8995911675
$1^3, -2, -1^2, -2, 3, -2, 3$	10^2_{138}		212	Hyperbolic	$Hyp1.91221025(0)$
$1^3, 2, 1^2, -3, 2^2, -3$	10^2_{169}		213	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[-11,32\|-1,3]$
$1^3, 2, -1^2, -3, 2^2, -3$	10^2_{163}		214	Non-geometric($2Hyp$)	Two-cusped $Hyp3.66386238(L208015)$, cusps truncated and identified
$1^3, -2, 1^2, 3, -2^2, 3$	10^2_{76}		215	Non-geometric($2Hyp$)	Two-cusped $Hyp6.13813879(L209044)$, cusps truncated and identified
$1^3, -2, -1^2, 3, -2^2, 3$	10^2_{162}		216	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[3,8\|2,5]$
$1^3, 2, -1, 2, 1, 3, -2, 3$	10^2_{135}		$\times$	Hyperbolic	Hyperbolic : 2.54158501007
$1^3, 2, -1, 2, -1, -3, 2, -3$	10^2_{155}		217	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp3.16396323(L106001)$
$1^3, -2, 1, -2, 1, 3, -2, 3$	10^2_{88}		218	Hyperbolic	Hyperbolic : 10.1788857787
$1^3, 2, 1, 3, -2, 1, 3, 2$	9^2_{45}	$\times$	$\times$	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(3,2)]$, $m = [-3,4\|-2,3]$

$\mathbf{x}$	L	π	M	T	D
$1^3, 2, 1, -3, 2, 1, -3, 2$	10^2_{128}	×	×	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(3,1)]$, $m = [-7, 8\| -6, 7]$
$1^3, 2, -1, 3, -2, 1, 3, -2$	9^2_{56}	×	×	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,-1)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^3, 2, -1, 3, -2, -1, 3, 2$	9^2_{47}	×	×	Seifert fibred	$SFS[T:(1,1)]$
$1^3, 2, -1, -3, 2, 1, -3, -2$	10^2_{160}	×	×	Graph manifold($2SFS$)	$SFS[D:(2,1)(3,-2)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^3, 2, -1, -3, 2, -1, -3, 2$	10^2_{154}		×	Non-geometric($1Hyp, 1SFS$)	$Hyp2.02988321(L104001) \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^3, -2, 1, 3, -2, 1, 3, -2$	10^2_{94}		219	Non-geometric($1Hyp, 2SFS$)	$Hyp2.02988321(L104001) \cup SFS[S^2 + 3\text{punctures}]$ $\cup SFS[A:(2,1)]$
$1^3, -2, 1, -3, 2, 1, -3, -2$	10^2_{124}	×	×	Graph manifold($1SFS$)	$SFS[A:(2,1)]/[-1, 8\|0, 1]$
$1^2, 2, -1^2, 2, 1, 3, -2, 3$	10^2_{137}		220	Hyperbolic	Hyperbolic : 5.2711507876
$1^2, -2, 1^2, -2, 1, 3, -2, 3$	10^2_{98}		221	Hyperbolic	Hyperbolic : 10.939571872
$1^2, 2, 1^2, -3, -2, 1, -2, -3$	10^2_{176}		222	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,-1)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2, 2, -1^2, -3, -2, 1, -2, -3$	9^2_{58}		223	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp2.82812209(L105002)$
$1^2, -2, 1^2, 3, 2, -1, 2, 3$	10^2_{177}		224	Hyperbolic	Hyperbolic : 6.78371351984
$1^2, -2, 1^2, 3, -2, 1, -2, 3$	10^2_{110}		225	Hyperbolic	Hyperbolic : 10.9022232311
$1^4, 2, -1, -3, 2, -3^2$	9^2_6		226	Hyperbolic	Hyperbolic : 4.6860342738
$1^4, -2, 1, 3, -2, 3^2$	10^2_{10}		227	Hyperbolic	Hyperbolic : 8.17462393347
$1^3, -2, 1^2, 3, -2, 3^2$	10^2_{42}		228	Hyperbolic	Hyperbolic : 9.31903694623
$1^3, -2, 1, -2^2, 3, -2, 3$	10^2_{18}		229	Hyperbolic	Hyperbolic : 7.7236288
$1^3, -2, 1, -2^2, -3, 2, -3$	9^2_9		230	Hyperbolic	Hyperbolic : 6.08964963846
$1^3, 2, -1, 2, 3, 2^2, 3$	9^2_{48}	×	×	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp2.02988321(L104001)$
$1^3, 2, -1, 2, -3, 2^2, -3$	9^2_{18}		231	Non-geometric($2Hyp$)	Two-cusped $Hyp5.33348957(L206003)$, cusps truncated and identified
$1^3, -2, 1, -2, 3, 2^2, 3$	10^2_{125}	×	×	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp2.82812209(L105002)$
$1^3, -2, 1, -2, 3, -2^2, 3$	10^2_{31}		232	Non-geometric($2Hyp$)	Two-cusped $Hyp6.59895154(v3426)$, cusps truncated and identified
$1^3, -2, 1, 3, -2^3, 3$	10^2_{63}		233	Hyperbolic	Hyperbolic : 7.92621498253
$1^3, 2^2, -1, 2, -3, 2, -3$	9^2_{26}		234	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp5.69302109$
$1^3, -2^2, 1, -2, 3, -2, 3$	10^2_{41}		235	Hyperbolic	Hyperbolic : 8.08678943364
$1^3, -2^2, 1, -2, -3, 2, -3$	9^2_8		236	Hyperbolic	Hyperbolic : 3.54091542134
$1^3, 2^2, 1, 3, 2^2, 3$	10^4_{12}	×	×	Seifert fibred	S^3
$1^3, 2^2, 1, 3, -2^2, 3$	10^4_{11}		×	Non-prime	$L(3,1)\sharp L(3,1)$
$1^3, 2^2, 1, -3, 2^2, -3$	10^4_{10}		×	Seifert fibred	$SFS[S^2:(3,1)(3,1)(3,-1)]$
$1^3, 2^2, -1, -3, 2^2, -3$	9^4_1		×	Seifert fibred	$SFS[S^2:(2,1)(2,1)(2,1)(3,-5)]$
$1^3, -2^2, 1, 3, -2^2, 3$	10^4_2		×	Seifert fibred	$SFS[S^2:(2,1)(5,1)(7,-5)]$
$1^3, 2^3, 1, 3, -2, 3$	9^2_{22}		237	Hyperbolic	Hyperbolic : 6.47082425363
$1^3, 2, -3, 2, -1, 2, -3, 2$	9^2_{38}		238	Non-geometric($1Hyp, 2SFS$)	$Hyp2.02988321(L104001)$ $\cup SFS[RP^2/n^2 + 2\text{punctures}:(1,1)]$ $\cup SFS[D:(2,1)(3,-2)]$
$1^3, -2, 3, -2, 1, -2, 3, -2$	10^2_{90}		239	Non-geometric($1Hyp, 2SFS$)	$Hyp2.82812209(L105002)$ $\cup SFS[RP^2/n^2 + 2\text{punctures}:(1,1)]$ $\cup SFS[D:(2,1)(3,-2)]$
$1^2, 2, -1^2, 2^2, 3, -2, 3$	9^2_{46}	×	×	Seifert fibred	$SFS[T:(2,1)]$
$1^2, 2, -1^2, 2^2, -3, 2, -3$	10^2_{129}	×	×	Non-geometric($2Hyp$)	Two-cusped $Hyp3.66386238(L208015)$, cusps truncated and identified
$1^2, -2, 1^2, -2^2, 3, -2, 3$	10^2_{35}		240	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp7.08492595$
$1^2, -2, 1^2, -2^2, -3, 2, -3$	9^2_{16}		241	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp4.40083252(L106002)$
$1^2, 2, 1^2, 2, 3, 2^2, 3$	10^4_A	×	×	Non-prime	$S^2 \times S^1 \sharp RP^3$
$1^2, 2, 1^2, 2, 3, -2^2, 3$	10^4_B	×	×	Non-prime	$RP^3 \sharp RP^3$
$1^2, 2, 1^2, 2, -3, 2^2, -3$	10^4_C	×	×	Seifert fibred	$SFS[S^2:(2,1)(2,1)(4,-3)]$
$1^2, 2, -1^2, 2, -3, 2^2, -3$	10^4_D	×	×	Seifert fibred	$SFS[S^2:(2,1)(4,1)(4,-3)]$
$1^2, -2, 1^2, -2, 3, -2^2, 3$	10^4_6		242	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp2.02988321(L104001)$
$1^2, 2, 1^2, -3, 2^3, -3$	10^2_{167}	×	×	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(5,2)]$, $m = [-7, 8\| -6, 7]$
$1^2, 2, 1^2, -3, -2^3, -3$	9^2_{60}		×	Seifert fibred	$SFS[S^2:(2,1)(2,1)(2,-1)]$
$1^2, -2, 1^2, 3, 2^3, 3$	9^2_{30}		243	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp5.69302109$
$1^2, -2, 1^2, 3, -2^3, 3$	10^2_{74}		244	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp6.44353738(L107005)$
$1^2, -2, 1, -2, 1, -2, 3, -2, 3$	10^2_{85}		245	Hyperbolic	Hyperbolic : 11.3777082973
$1^2, 2, -1, 2, 1, 3, -2^2, 3$	10^2_{170}		×	Non-geometric($2Hyp$)	Two-cusped $Hyp3.66386238(L205001)$, cusps truncated and identified
$1^2, -2, 1, -2, 1, 3, 2^2, 3$	10^2_{172}		×	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp5.69302109$
$1^2, -2, 1, -2, 1, 3, -2^2, 3$	10^2_{107}		246	Non-geometric($2Hyp$)	Two-cusped $Hyp8.99735194$, cusps truncated and identified
$1^2, -2, 1, -2, 1, -3, 2^2, -3$	10^2_{174}		×	Seifert fibred	$SFS[T:(1,1)]$
$1^2, 2, -1, 2^2, 1, 3, -2, 3$	10^2_{156}		×	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp4.40083252(L106002)$
$1^2, -2, 1, -2^2, 1, 3, -2, 3$	10^2_{101}		247	Hyperbolic	Hyperbolic : 9.96636799677
$1^2, 2, -1, 2, 3, 2, -1, 2, 3$	10^2_{179}		×	Non-prime	$L(3,1)\sharp L(3,1)$
$1^2, 2, -1, 2, -3, 2, -1, 2, -3$	10^2_{175}		×	Hyperbolic	Hyperbolic : 3.3849452212

$\mathbf{x}$	L	π	M	T	D
$1^2, -2, 1, -2, 3, -2, 1, -2, 3$	10^2_{118}		248	Hyperbolic	Hyperbolic : 11.4996508049
$1^2, -2, 1, -2, -3, -2, 1, -2, -3$	10^2_{183}		249	Hyperbolic	Hyperbolic : 5.21836247903
$1^2, -2, 1, -2, 3, -2, 1, 3, -2$	10^2_{93}		250	Non-geometric($1Hyp, 2SFS$)	Two-cusped $Hyp3.66386238(L205001)$ $\cup SFS[S^2 + 3\text{punctures}] \cup SFS[D : (2,1)(3,-2)]$
$1^2, -2, 1, -2, -3, 2, 1, -3, -2$	9^2_{36}		251	Graph manifold($3SFS$)	$SFS[D : (2,1)(3,-1)] \cup Graph[S^2 + 3\text{punctures}]$ $\cup SFS[A : (2,1)]$
$1^2, -2, 1, 3, 2, -1, 2^2, 3$	10^2_{142}		252	Hyperbolic	Hyperbolic : 6.70616178698
$1^2, -2, 1, 3, -2, 1, -2^2, 3$	10^2_{108}		253	Hyperbolic	Hyperbolic : 11.3366219944
$1^2, -2, 1, 3, -2, 1, -2, 3, -2$	10^2_{117}		254	Hyperbolic	Hyperbolic : 11.6906301719
$1^2, -2, 1, 3, -2, 1, 3, -2^2$	10^2_{97}		255	Hyperbolic	Hyperbolic : 11.4950887093
$1^2, 2, -1, 3, 2^2, 1, 3, -2$	10^4_{15}		256	Seifert fibred	$SFS[S^2 : (2,1)(4,1)(4,-3)]$
$1^2, 2, -1, -3, 2^2, 1, -3, -2$	10^4_{13}	$\times$	$\times$	Seifert fibred	$SFS[S^2 : (2,1)(4,1)(4,-3)]$
$1^2, -2, 1, 3, 2^2, 1, 3, -2$	10^4_{19}		257	Graph manifold($2SFS$)	$SFS[D : (2,1)(2,1)] \cup /mSFS[D : (2,1)(3,1)]$, $m = [-1, 2 \mid 0, 1]$
$1^2, -2, 1, 3, -2^2, 1, 3, -2$	10^4_8		258	Graph manifold($2SFS$)	$SFS[D : (2,1)(3,-2)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2, -2, 1, -3, 2^2, 1, -3, -2$	10^4_{20}		259	Non-prime	$S^3/P48 \sharp S^2 \times S^1$
$1^2, 2^2, 1^2, 2, -3, 2, -3$	10^2_{165}		$\times$	Seifert fibred	$SFS[S^2 : (2,1)(2,1)(6,-5)]$
$1^2, -2^2, 1^2, -2, 3, -2, 3$	10^2_{72}		260	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp7.64337517$
$1^2, -2^2, 1^2, -2, -3, 2, -3$	9^2_{17}		261	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp3.16396323(L106001)$
$1^2, 2^2, 1^2, 3, 2^2, 3$	10^4_{16}		262	Non-prime	$RP^3 \sharp RP^3$
$1^2, 2^2, 1^2, 3, -2^2, 3$	10^4_{14}		$\times$	Graph manifold($2SFS$)	$SFS[D : (2,1)(2,1)] \cup /mSFS[D : (2,1)(3,1)]$, $m = [-1, 2 \mid 0, 1]$
$1^2, 2^2, 1^2, -3, 2^2, -3$	10^4_{17}	$\times$	$\times$	Seifert fibred	$SFS[S^2 : (2,1)(2,1)(4,-3)]$
$1^2, -2^2, 1^2, 3, -2^2, 3$	10^4_4		$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp2.02988321(L104001)$
$1, -2, 1, -2, 1, -2, 1, 3, -2, 3$	10^2_{114}		263	Hyperbolic	Hyperbolic : 12.8063275482
$1, -2, 1, -2, 1, -2, 1, -3, 2, -3$	10^2_{180}		264	Hyperbolic	Hyperbolic : 5.78328456214
$1, 2, -1, 2, 1, 3, -2, 1, -2, 3$	10^2_{181}		265	Hyperbolic	Hyperbolic : 7.90216370907
$1, 2, -1, 2, 1, -3, -2, 1, -2, -3$	10^2_{178}		266	Graph manifold($2SFS$)	$SFS[D : (2,1)(2,1)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1, -2, 1, -2, 1, 3, -2, 1, -2, 3$	10^2_{120}		267	Hyperbolic	Hyperbolic : 13.7501824593
$1, -2, 1, -2, 1, -3, -2, 1, -2, -3$	10^2_{184}		268	Hyperbolic	Hyperbolic : 9.8481495022
$1^3, -2, 1, -2, 3, -2, 3^2$	10^2_{19}		269	Hyperbolic	Hyperbolic : 9.22966818954
$1^3, 2, -1, -3, 2^2, -3^2$	9^2_{28}		270	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp4.40083252(L106002)$
$1^3, -2, 1, 3, -2^2, 3^2$	10^2_{64}		271	Hyperbolic	Hyperbolic : 8.65334159139
$1^3, 2, -1, -3, 2, -3, 2, -3$	9^2_{33}		272	Non-geometric($1Hyp, 1SFS$)	$Hyp2.82812209(L105002) \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^3, -2, 1, 3, -2, 3, -2, 3$	10^2_{81}		273	Non-geometric($1Hyp, 1SFS$)	$Hyp4.40083252 \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2, 2, 1^2, 2, -3, 2, -3^2$	10^2_{130}	$\times$	$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp2.02988321(L104001)$
$1^2, 2, -1^2, 2, -3, 2, -3^2$	10^2_{131}	$\times$	$\times$	Non-geometric($2Hyp$)	Two-cusped $Hyp3.66386238(L205001)$, cusps truncated and identified
$1^2, -2, 1^2, -2, 3, -2, 3^2$	10^2_{37}		$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp7.64337517$
$1^2, 2, 1^2, -3, 2^2, -3^2$	10^2_{168}	$\times$	$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp2.82812209(L105002)$
$1^2, 2, -1^2, -3, 2^2, -3^2$	10^2_{161}	$\times$	$\times$	Graph manifold($2SFS$)	$SFS[D : (2,1)(3,-2)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2, -2, 1^2, 3, -2^2, 3^2$	10^2_{75}		$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp7.08492595(L107006)$
$1^2, 2, 1^2, -3, 2, -3, 2, -3$	10^4_{18}	$\times$	$\times$	Seifert fibred	$SFS[T : (1,1)]$
$1^2, -2, 1^2, 3, -2, 3, -2, 3$	10^4_7		$\times$	Graph manifold($2SFS$)	$SFS[D : (2,1)(3,-2)] \cup Graph[S^2 + 3\text{punctures}]$ /[non-fibre-preserving gluing]
$1^2, -2, 1, -2, 1, 3, -2, 3^2$	10^2_{86}		274	Hyperbolic	Hyperbolic : 11.711992114
$1^2, -2, 1, 3, -2, 1, 3, -2, 3$	10^2_{119}		275	Hyperbolic	Hyperbolic : 13.0919974636
$1^2, -2, 1, 3, -2, 1, 3^2, -2$	10^2_{91}		276	Non-geometric($1Hyp, 2SFS$)	Two-cusped $Hyp3.66386238(L208015)$ $\cup SFS[S^2 + 3\text{punctures}] \cup SFS[D : (2,1)(3,-2)]$
$1^3, -2, 1, 3, -2, 3^3$	10^2_{17}		277	Hyperbolic	Hyperbolic : 7.5176899
$1^3, -2, 1, 3^2, -2, 3^2$	10^2_{30}		278	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(2,-1)] \cup Hyp5.1379412(L107004)$
$1^2, -2, 1^2, 3^2, -2, 3^2$	10^4_5		279	Graph manifold($1SFS$)	$SFS[A : (2,1)]/[-1, 6 \mid 0, 1]$
$1^3, 2, -1, -3, 2, 4, -3, 4$	9_{15}		280	Hyperbolic	Hyperbolic : 8.70500592768
$1^3, -2, 1, 3, -2, -4, 3, -4$	10_{29}		281	Hyperbolic	Hyperbolic : 10.8352925121
$1^2, 2, 1^2, -3, 2, 4, -3, 4$	10^3_{49}	$\times$	$\times$	Graph manifold($2SFS$)	$SFS[D : (2,1)(2,1)] \cup /mSFS[D : (2,1)(3,2)]$, $m = [0, 1 \mid 1, 0]$
$1^2, 2, -1^2, -3, 2, 4, -3, 4$	10^3_{48}	$\times$	$\times$	Graph manifold($2SFS$)	$SFS[D : (2,1)(2,1)] \cup /mSFS[D : (2,1)(3,1)]$, $m = [0, 1 \mid 1, 0]$
$1^2, -2, 1^2, 3, -2, -4, 3, -4$	10^3_5		282	Hyperbolic	Hyperbolic : 5.50474883782
$1^2, -2, 1, -2^3, 3, -2, 3$	10^2_{26}		283	Hyperbolic	Hyperbolic : 7.9158333
$1^2, 2, -1, 2^2, -3, 2^2, -3$	9^2_{15}		$\times$	Non-geometric($2Hyp$)	Two-cusped $Hyp4.05976643(L206002)$, cusps truncated and identified
$1^2, -2, 1, -2^2, 3, -2^2, 3$	10^2_{36}		$\times$	Non-geometric($2Hyp$)	Two-cusped $Hyp6.59895154(v3426)$, cusps truncated and identified
$1^2, -2, 1, 3, -2^4, 3$	10^2_{62}		284	Hyperbolic	Hyperbolic : 8.15532021622
$1^2, -2^2, 1, -2^2, 3, -2, 3$	10^2_{43}		285	Hyperbolic	Hyperbolic : 9.98319303033
$1^2, -2^2, 1, -2, 3, -2^2, 3$	10^2_{79}		286	Non-geometric($2Hyp$)	Two-cusped $Hyp7.7069118(L210076)$, cusps truncated and identified

$\mathbf{x}$	L	π	M	T	D
$1^2, 2^2, 1, 3, 2^3, 3$	10^2_{149}		287	Graph manifold($2SFS$)	$SFS[D:(2,1)(2,1)] \cup /mSFS[D:(2,1)(3,2)]$, $m = [-3,4\|-2,3]$
$1^2, 2^2, 1, 3, -2^3, 3$	10^2_{144}		$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp3.16396323(L106001)$
$1^2, 2^2, 1, -3, 2^3, -3$	10^2_{143}		$\times$	Graph manifold($1SFS$)	$SFS[A:(3,1)]/[-1,12\|0,1]$
$1^2, 2^2, 1, -3, -2^3, -3$	10^2_{150}		$\times$	Non-prime	$S^2 \times S^1 \sharp S^2 \times S^1$
$1^2, -2^2, 1, 3, 2^3, 3$	10^2_{145}		$\times$	Non-geometric($2Hyp$)	Two-cusped $Hyp5.33348957(L209045)$, cusps truncated and identified
$1^2, -2^2, 1, 3, -2^3, 3$	10^2_{50}		$\times$	Non-geometric($2Hyp$)	Two-cusped $Hyp6.59895154(v3426)$, cusps truncated and identified
$1^2, -2^3, 1, -2, 3, -2, 3$	10^2_{48}		288	Hyperbolic	Hyperbolic : 9.29239133504
$1, -2, 1, -2, 1, -2^2, 3, -2, 3$	10^2_{83}		289	Non-geometric($1Hyp, 1SFS$)	$Hyp4.40083252(L106002) \cup Graph[S^2 + 3$punctures$]$ /[non-fibre-preserving gluing]
$1, -2, 1, -2, 1, -2^2, -3, 2, -3$	9^2_{32}		$\times$	Non-geometric($1Hyp, 1SFS$)	$Hyp2.82812209(L105002) \cup Graph[S^2 + 3$punctures$]$ /[non-fibre-preserving gluing]
$1, -2, 1, -2, 1, 3, -2^3, 3$	10^2_{104}		$\times$	Non-geometric($1Hyp, 1SFS$)	$Hyp4.40083252 \cup Graph[S^2 + 3$punctures$]$ /[non-fibre-preserving gluing]
$1, -2, 1, -2, 1, -3, -2^3, -3$	10^2_A		$\times$	Graph manifold($2SFS$)	$SFS[D:(2,1)(5,-3)] \cup Graph[S^2 + 3$punctures$]$ /[non-fibre-preserving gluing]
$1, -2, 1, -2^2, 1, 3, 2^2, 3$	10^2_{171}		$\times$	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp4.40083252(L106002)$
$1, -2, 1, -2^2, 1, 3, -2^2, 3$	10^2_{106}		$\times$	Non-geometric($2Hyp$)	Two-cusped $Hyp8.99735194(L207006)$, cusps truncated and identified
$1, -2, 1, -2^2, 1, -3, 2^2, -3$	10^2_{173}		$\times$	Seifert fibred	$SFS[T:(1,1)]$
$1, -2, 1, -2^3, 1, 3, -2, 3$	10^2_{87}		290	Hyperbolic	Hyperbolic : 9.96152429801
$1, 2, -1, 2^2, 3, -2, 1, -2, 3$	10^2_{158}		291	Seifert fibred	$SFS[T:(3,1)]$
$1, -2, 1, -2^2, 3, -2, 1, -2, 3$	10^2_{115}		292	Hyperbolic	Hyperbolic : 11.3550479541
$1, -2, 1, -2, 3, -2, 1, -2^2, 3$	10^2_{116}		293	Hyperbolic	Hyperbolic : 13.0579257203
$1, -2, 1, 3, -2^2, 1, -2^2, 3$	10^2_{109}		294	Hyperbolic	Hyperbolic : 11.6973077174
$1^2, -2, 1, -2^2, 3, -2, 3^2$	10^2_{27}		295	Hyperbolic	Hyperbolic : 9.53187984
$1^2, -2, 1, -2, 3, -2^2, 3^2$	10^2_{45}		296	Hyperbolic	Hyperbolic : 10.336314
$1^2, -2, 1, -2, 3, -2, 3, -2, 3$	10^2_{84}		297	Non-geometric($1Hyp, 1SFS$)	$Hyp5.69302109 \cup Graph[S^2 + 3$punctures$]$ /[non-fibre-preserving gluing]
$1^2, -2, 1, 3, 2^3, 3^2$	9^2_{24}		298	Hyperbolic	Hyperbolic : 8.92573094642
$1^2, -2, 1, 3, -2^3, 3^2$	10^2_{66}		299	Hyperbolic	Hyperbolic : 8.84596467061
$1^2, 2^2, 1, 3, 2^2, 3^2$	10^2_{151}		300	Hyperbolic	Hyperbolic : 6.3545865572
$1^2, 2^2, 1, 3, -2^2, 3^2$	10^2_{148}		301	Hyperbolic	Hyperbolic : 8.9293178231
$1^2, 2^2, 1, -3, 2^2, -3^2$	10^2_{147}		302	Hyperbolic	Hyperbolic : 8.87426862421
$1^2, 2^2, 1, -3, -2^2, -3^2$	10^2_{152}		303	Hyperbolic	Hyperbolic : 6.08964963846
$1^2, -2^2, 1, 3, -2^2, 3^2$	10^2_{54}		304	Hyperbolic	Hyperbolic : 11.4721387075
$1^2, -2^2, 1, 3, -2, 3, -2, 3$	10^2_{105}		$\times$	Non-geometric($1Hyp, 1SFS$)	$Hyp5.69302109 \cup Graph[S^2 + 3$punctures$]$ /[non-fibre-preserving gluing]
$1^2, -2, 3, -2, 1, -2, 3^2, -2$	10^2_{96}		305	Non-geometric($2Hyp, 1SFS$)	$Hyp2.02988321(L104001)$ $\cup SFS[RP^2 + 2$punctures$/n^2:(1,1)]$ $\cup Hyp2.02988321(L104001)$
$1, -2, 1, -2, 1, 3, -2, 3, -2, 3$	10^4_9		$\times$	Graph manifold($2SFS$)	$Graph[S^2 + 3$punctures$] \cup Graph[S^2 + 3$punctures$]$
$1, -2, 1, -2, 1, -3, 2, -3, 2, -3$	10^4_{21}		306	Non-prime	$T \times S^1 \sharp S^2 \times S^1$
$1, -2, 1, -2, 3, -2, 1, 3, -2, 3$	10^2_{121}		307	Hyperbolic	Hyperbolic : 13.564606056
$1^2, 2, -1, 2, 3, -2, -4, 3, -4$	8_1		308	Graph manifold($1SFS$)	$SFS[A:(3,1)]/[0,1\|1,-2]$
$1^2, 2, -1, 2, -3, 2, 4, -3, 4$	9_{21}		309	Hyperbolic	Hyperbolic : 8.98396582335
$1^2, -2, 1, -2, 3, -2, -4, 3, -4$	10_{42}		310	Hyperbolic	Hyperbolic : 12.2427702807
$1^2, -2, 1, -2, -3, 2, 4, -3, 4$	9_8		311	Hyperbolic	Hyperbolic : 7.46191985006
$1^2, -2, 1, 3, 2^2, -4, 3, -4$	9_{25}		312	Hyperbolic	Hyperbolic : 10.3008554297
$1^2, -2, 1, 3, -2^2, -4, 3, -4$	10_{71}		313	Hyperbolic	Hyperbolic : 12.3609050663
$1, -2, 1, -2, 1, 3, -2, -4, 3, -4$	10^3_{29}		314	Non-geometric($2Hyp$)	Two-cusped $Hyp5.33348957(L209045)$, cusps truncated and identified
$1, -2, 1, -2, 1, -3, 2, 4, -3, 4$	10^3_{61}	$\times$	$\times$	Seifert fibred	$KB/n^2 \tilde{\times} S^1$
$1^2, 2, -1, -3, 2, -3, 4, -3, 4$	9_{14}		315	Hyperbolic	Hyperbolic : 7.67266968706
$1^2, 2, -1, -3, 2, -3, -4, 3, -4$	8_3		316	Graph manifold($2SFS$)	$SFS[A:(2,1)] \cup SFS[A:(2,-1)]$
$1^2, -2, 1, 3, -2, 3, 4, -3, 4$	9_{12}		317	Hyperbolic	Hyperbolic : 7.63491871137
$1^2, -2, 1, 3, -2, 3, -4, 3, -4$	10_{44}		318	Hyperbolic	Hyperbolic : 12.2807182195
$1^2, -2, 1, 3, -2, -4, 3^2, -4$	10^3_{12}		$\times$	Hyperbolic	Hyperbolic : 6.18027441937
$1^2, -2, 1, 3, -2, -4, 3, -4^2$	10_{43}		319	Hyperbolic	Hyperbolic : 11.5029344901
$1, -2, 1, -2^4, 3, -2, 3$	10^2_{25}		320	Hyperbolic	Hyperbolic : 7.94057924778
$1, -2, 1, -2^3, 3, -2^2, 3$	10^2_{34}		$\times$	Non-geometric($1Hyp$)	Two-cusped $Hyp4.74949998(L207001)$, cusps truncated and identified
$1, -2, 1, -2, 3, -2^4, 3$	10^2_{39}		321	Hyperbolic	Hyperbolic : 8.56937006859
$1, -2, 1, 3, -2^5, 3$	10^2_{55}		322	Non-geometric($1Hyp, 1SFS$)	$SFS[D:(2,1)(2,-1)] \cup Hyp3.33174423(L107002)$
$1, -2^2, 1, -2, 3, -2^3, 3$	10^2_{78}		$\times$	Non-geometric($1Hyp$)	Two-cusped $Hyp6.13813879(L210120)$, cusps truncated and identified
$1, -2^2, 1, 3, -2^4, 3$	10^4_1		$\times$	Hyperbolic	$Hyp2.25976713(0)$
$1, 2^3, 1, -3, 2^3, -3$	10^2_{146}		323	Hyperbolic	Hyperbolic : 8.50516589391
$1, -2^3, 1, 3, -2^3, 3$	10^2_{52}		324	Hyperbolic	Hyperbolic : 11.2316198792
$1, -2^3, 3, -2, 1, -2, 3, -2$	10^2_{100}		325	Hyperbolic	Hyperbolic : 9.11455084582
$1, -2^2, 3, -2, 1, -2^2, 3, -2$	10^2_{103}		326	Hyperbolic	Hyperbolic : 11.8229082126

$\mathbf{x}$	L	π	M	T	D
$1, -2^2, 3, -2, 1, -2, 3, -2^2$	10^2_{112}		327	Hyperbolic	Hyperbolic : 10.860405945
$1, -2, 1, -2^2, 3, -2, -4, 3, -4$	10_{41}		328	Hyperbolic	Hyperbolic : 11.6510447
$1, -2, 1, -2^2, -3, 2, 4, -3, 4$	9_{19}		329	Hyperbolic	Hyperbolic : 8.73622201206
$1, -2, 1, -2, 3, 2^2, -4, 3, -4$	10_{137}		330	Hyperbolic	Hyperbolic : 7.51768989647
$1, -2, 1, -2, 3, -2^2, -4, 3, -4$	10_{59}		331	Hyperbolic	Hyperbolic : 12.6232644507
$1, -2, 1, -2, -3, 2^2, 4, -3, 4$	10_{136}		332	Hyperbolic	Hyperbolic : 6.55174328789
$1, -2, 1, -2, -3, -2^2, 4, -3, 4$	10_{138}		333	Hyperbolic	Hyperbolic : 9.70163046606
$1, -2, 1, 3, 2^3, -4, 3, -4$	10_{54}		334	Hyperbolic	Hyperbolic : 10.3008554297
$1, -2, 1, 3, -2^3, -4, 3, -4$	10_{70}		335	Hyperbolic	Hyperbolic : 11.7321635905
$1, -2^2, 1, -2, 3, -2, -4, 3, -4$	10^3_6		336	Hyperbolic	Hyperbolic : 5.62938237498
$1, -2^2, 1, -2, -3, 2, 4, -3, 4$	9^3_7		337	Seifert fibred	$SFS[S^2 : (2,1)(4,1)(9,-7)]$
$1, 2^2, 1, 3, -2^2, -4, 3, -4$	10^3_{53}		338	Hyperbolic	$Hyp1.94150308(Z_2)$
$1, -2^2, 1, 3, -2^2, -4, 3, -4$	10^3_{10}		339	Hyperbolic	Hyperbolic : 5.88829048381
$1, -2^2, 1, -3, -2^2, 4, -3, 4$	10^3_{54}		340	Hyperbolic	Hyperbolic : 3.38319789365
$1, -2, 3, -2, 1, -2, -4, 3, -2, -4$	10^3_{37}		341	Non-geometric($1Hyp, 1SFS$)	$SFS[D : (2,1)(3,-2)] \cup Hyp2.82812209(L105002)$
$1, -2, -3, -2, 1, -2, 4, -3, -2, 4$	10^3_{59}	$\times$	$\times$	Graph manifold($2SFS$)	$SFS[D : (2,1)(3,1)] \cup /mSFS[D : (2,1)(3,1)]$, $m = [-7, 8\vert -6, 7]$
$1, -2, 1, -2, 3, -2, 3, -4, 3, -4$	10_{45}		342	Hyperbolic	Hyperbolic : 12.7549233603
$1, -2, 1, -2, 3, -2, -4, 3^2, -4$	10^3_{13}		$\times$	Hyperbolic	Hyperbolic : 6.3326666425
$1, -2, 1, 3, -2^2, -4, 3^2, -4$	10^3_{24}		343	Non-geometric($1Hyp, 1SFS$)	$SFS[\mathrm{Or}, g = 1 + 1puncture : (1,2)]$ $\cup Hyp2.02988321(L104001)$
$1, -2, 1, 3, -2, 3, -2, -4, 3, -4$	10_{88}		344	Hyperbolic	Hyperbolic : 14.6641288398
$1, -2, 1, 3, -2, -4, 3, -2, -4, 3$	10^3_{34}		345	Non-geometric($2Hyp$)	$Hyp2.02988321(L104001) \cup Hyp2.02988321(L104001)$
$1, -2, 1, 3, -2, -4, 3, 5, -4, 5$	10^2_{23}		346	Hyperbolic	Hyperbolic : 9.06406948946

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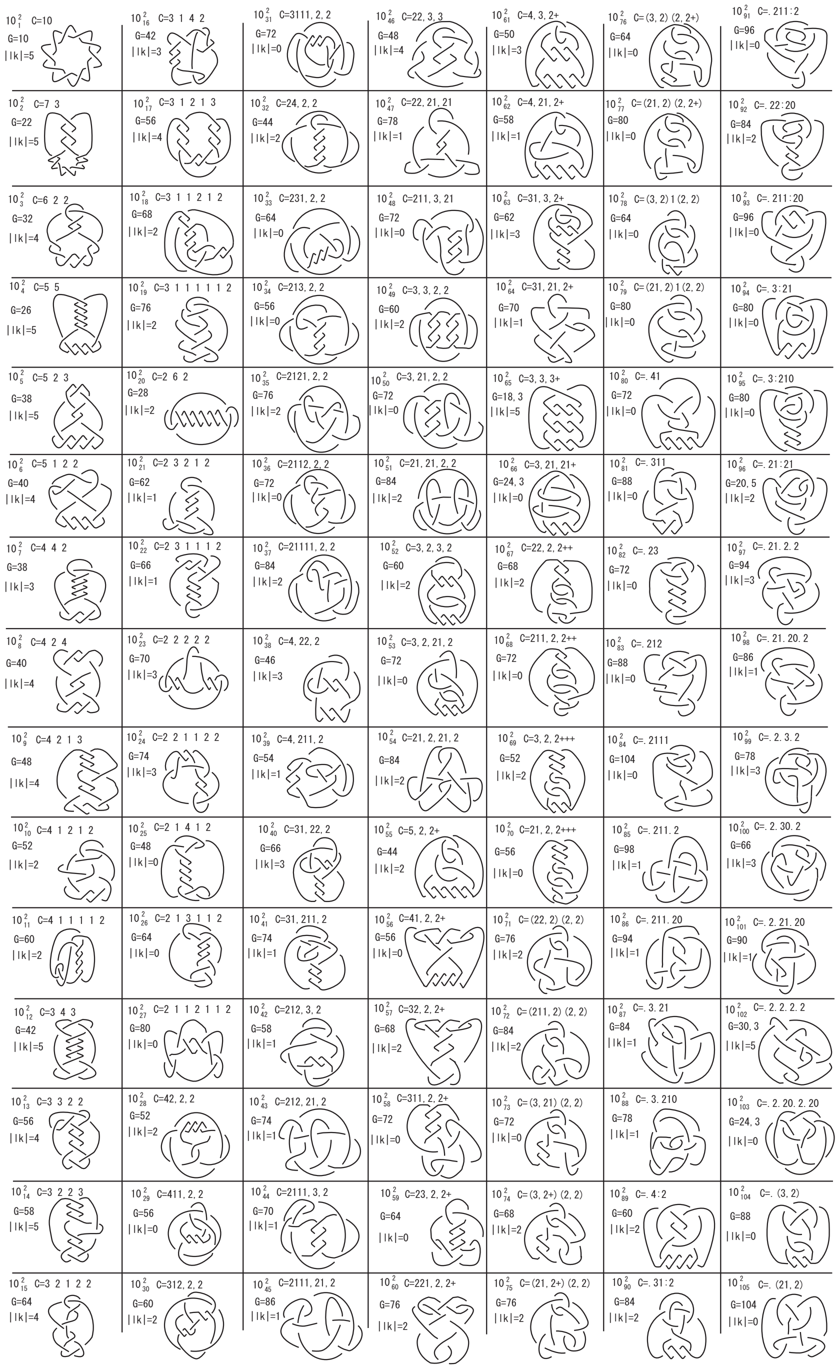
Appendix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10^2_{9} C=4 2 1 3 G=48 |lk|=4
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10^2_{11} C=4 1 1 1 1 2 G=60 |lk|=2
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10^2_{19} C=3 1 1 1 1 1 2 G=76 |lk|=2
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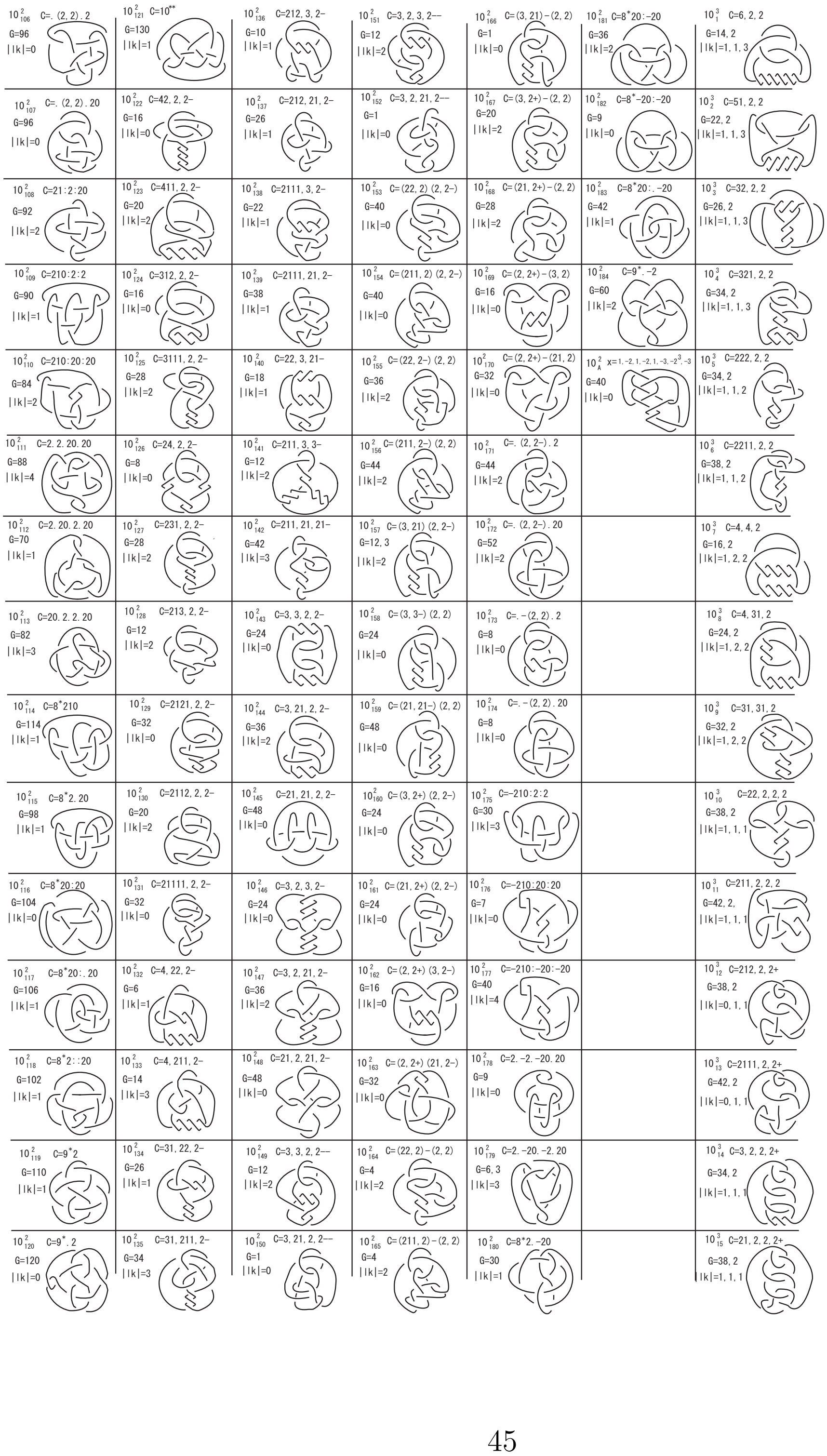

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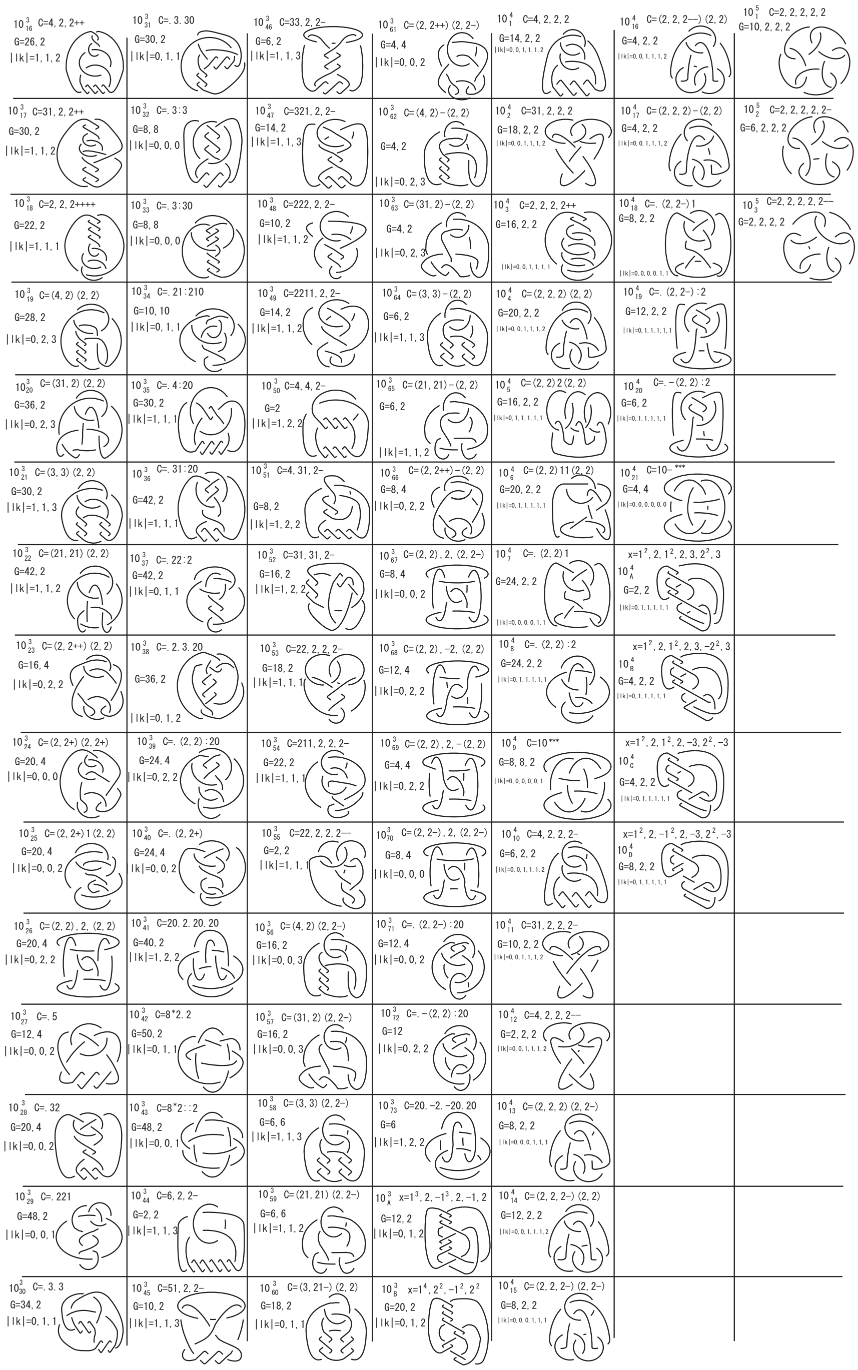
10 3 12 C=212, 2, 2+ G=38, 2 |lk|=0, 1, 1
10 3 13 C=2111, 2, 2+ G=42, 2 |lk|=0, 1, 1
10 3 14 C=3, 2, 2, 2+ G=34, 2 |lk|=1, 1, 1
10 3 15 C=21, 2, 2, 2+ G=38, 2 |lk|=1, 1, 1



10 3 16 C=4, 2, 2++ G=26, 2 |lk|=1, 1, 2
10 3 17 C=31, 2, 2++ G=30, 2 |lk|=1, 1, 2
10 3 18 C=2, 2, 2++++ G=22, 2 |lk|=1, 1, 1
10 3 19 C=(4, 2) (2, 2) G=28, 2 |lk|=0, 2, 3
10 3 20 C=(31, 2) (2, 2) G=36, 2 |lk|=0, 2, 3
10 3 21 C=(3, 3) (2, 2) G=30, 2 |lk|=1, 1, 3
10 3 22 C=(21, 21) (2, 2) G=42, 2 |lk|=1, 1, 2
10 3 23 C=(2, 2++) (2, 2) G=16, 4 |lk|=0, 2, 2
10 3 24 C=(2, 2+) (2, 2+) G=20, 4 |lk|=0, 0, 0
10 3 25 C=(2, 2+) 1 (2, 2) G=20, 4 |lk|=0, 0, 2
10 3 26 C=(2, 2), 2, (2, 2) G=20, 4 |lk|=0, 2, 2
10 3 27 C=. 5 G=12, 4 |lk|=0, 0, 2
10 3 28 C=. 32 G=20, 4 |lk|=0, 0, 2
10 3 29 C=. 221 G=48, 2 |lk|=0, 0, 1
10 3 30 C=. 3. 3 G=34, 2 |lk|=0, 1, 1
10 3 31 C=. 3. 30 G=30, 2 |lk|=0, 1, 1
10 3 32 C=. 3:3 G=8, 8 |lk|=0, 0, 0
10 3 33 C=. 3:30 G=8, 8 |lk|=0, 0, 0
10 3 34 C=. 21:210 G=10, 10 |lk|=0, 1, 1
10 3 35 C=. 4:20 G=30, 2 |lk|=1, 1, 1
10 3 36 C=. 31:20 G=42, 2 |lk|=1, 1, 1
10 3 37 C=. 22:2 G=42, 2 |lk|=0, 1, 1
10 3 38 C=. 2. 3. 20 G=36, 2 |lk|=0, 1, 2
10 3 39 C=. (2, 2):20 G=24, 4 |lk|=0, 2, 2
10 3 40 C=. (2, 2+) G=24, 4 |lk|=0, 0, 2
10 3 41 C=20. 2. 20. 20 G=40, 2 |lk|=1, 2, 2
10 3 42 C=8*2. 2 G=50, 2 |lk|=0, 1, 1
10 3 43 C=8*2::2 G=48, 2 |lk|=0, 0, 1
10 3 44 C=6, 2, 2− G=2, 2 |lk|=1, 1, 3
10 3 45 C=51, 2, 2− G=10, 2 |lk|=1, 1, 3
10 3 46 C=33, 2, 2− G=6, 2 |lk|=1, 1, 3
10 3 47 C=321, 2, 2− G=14, 2 |lk|=1, 1, 3
10 3 48 C=222, 2, 2− G=10, 2 |lk|=1, 1, 2
10 3 49 C=2211, 2, 2− G=14, 2 |lk|=1, 1, 2
10 3 50 C=4, 4, 2− G=2 |lk|=1, 2, 2
10 3 51 C=4, 31, 2− G=8, 2 |lk|=1, 2, 2
10 3 52 C=31, 31, 2− G=16, 2 |lk|=1, 2, 2
10 3 53 C=22, 2, 2, 2− G=18, 2 |lk|=1, 1, 1
10 3 54 C=211, 2, 2, 2− G=22, 2 |lk|=1, 1, 1
10 3 55 C=22, 2, 2, 2−− G=2, 2 |lk|=1, 1, 1
10 3 56 C=(4, 2) (2, 2−) G=16, 2 |lk|=0, 0, 3
10 3 57 C=(31, 2) (2, 2−) G=16, 2 |lk|=0, 0, 3
10 3 58 C=(3, 3) (2, 2−) G=6, 6 |lk|=1, 1, 3
10 3 59 C=(21, 21) (2, 2−) G=6, 6 |lk|=1, 1, 2
10 3 60 C=(3, 21−) (2, 2) G=18, 2 |lk|=0, 1, 1
10 3 61 C=(2, 2++) (2, 2−) G=4, 4 |lk|=0, 0, 2
10 3 62 C=(4, 2)−(2, 2) G=4, 2 |lk|=0, 2, 3
10 3 63 C=(31, 2)−(2, 2) G=4, 2 |lk|=0, 2, 3
10 3 64 C=(3, 3)−(2, 2) G=6, 2 |lk|=1, 1, 3
10 3 65 C=(21, 21)−(2, 2) G=6, 2 |lk|=1, 1, 2
10 3 66 C=(2, 2++)−(2, 2) G=8, 4 |lk|=0, 2, 2
10 3 67 C=(2, 2), 2, (2, 2−) G=8, 4 |lk|=0, 0, 2
10 3 68 C=(2, 2), −2, (2, 2) G=12, 4 |lk|=0, 2, 2
10 3 69 C=(2, 2), 2, −(2, 2) G=4, 4 |lk|=0, 2, 2
10 3 70 C=(2, 2−), 2, (2, 2−) G=8, 4 |lk|=0, 0, 0
10 3 71 C=. (2, 2−):20 G=12, 4 |lk|=0, 0, 2
10 3 72 C=. −(2, 2):20 G=12 |lk|=0, 2, 2
10 3 73 C=20. −2. −20. 20 G=6 |lk|=1, 2, 2
10 3 A x=1³, 2, −1³, 2, −1, 2 G=12, 2 |lk|=0, 1, 2
10 3 B x=1⁴, 2², −1², 2² G=20, 2 |lk|=0, 1, 2
10 4 1 C=4, 2, 2, 2 G=14, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 2 C=31, 2, 2, 2 G=18, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 3 C=2, 2, 2, 2++ G=16, 2, 2 |lk|=0, 0, 1, 1, 1, 1
10 4 4 C=(2, 2, 2) (2, 2) G=20, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 5 C=(2, 2) 2 (2, 2) G=16, 2, 2 |lk|=0, 1, 1, 1, 1, 1
10 4 6 C=(2, 2) 11 (2, 2) G=20, 2, 2 |lk|=0, 1, 1, 1, 1, 1
10 4 7 C=. (2, 2) 1 G=24, 2, 2 |lk|=0, 0, 0, 0, 1, 1
10 4 8 C=. (2, 2):2 G=24, 2, 2 |lk|=0, 1, 1, 1, 1, 1
10 4 9 C=10*** G=8, 8, 2 |lk|=0, 0, 0, 0, 0, 1
10 4 10 C=4, 2, 2, 2− G=6, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 11 C=31, 2, 2, 2− G=10, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 12 C=4, 2, 2, 2−− G=2, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 13 C=(2, 2, 2) (2, 2−) G=8, 2, 2 |lk|=0, 0, 0, 1, 1, 1
10 4 14 C=(2, 2, 2−) (2, 2) G=12, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 15 C=(2, 2, 2−) (2, 2−) G=8, 2, 2 |lk|=0, 0, 0, 1, 1, 1
10 4 16 C=(2, 2, 2−−) (2, 2) G=4, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 17 C=(2, 2, 2)−(2, 2) G=4, 2, 2 |lk|=0, 0, 1, 1, 1, 2
10 4 18 C=. (2, 2−) 1 G=8, 2, 2 |lk|=0, 0, 0, 0, 1, 1
10 4 19 C=. (2, 2−):2 G=12, 2, 2 |lk|=0, 1, 1, 1, 1, 1
10 4 20 C=. −(2, 2):2 G=6, 2 |lk|=0, 1, 1, 1, 1, 1
10 4 21 C=10−*** G=4, 4 |lk|=0, 0, 0, 0, 0, 0
x=1², 2, 1², 2, 3, 2², 3 10 4 A G=2, 2 |lk|=0, 1, 1, 1, 1, 1
x=1², 2, 1², 2, 3, −2², 3 10 4 B G=4, 2, 2 |lk|=0, 1, 1, 1, 1, 1
x=1², 2, 1², 2, −3, 2², −3 10 4 C G=4, 2, 2 |lk|=0, 1, 1, 1, 1, 1
x=1², 2, −1², 2, −3, 2², −3 10 4 D G=8, 2, 2 |lk|=0, 1, 1, 1, 1, 1
10 5 1 C=2, 2, 2, 2, 2 G=10, 2, 2, 2
10 5 2 C=2, 2, 2, 2, 2− G=6, 2, 2, 2
10 5 3 C=2, 2, 2, 2, 2−− G=2, 2, 2, 2