Chaper: A model for mind relations

- An application of knot theory to psychology *

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ABSTRACT

Evaluating the five-factors in Five-Factor Model of personality established by H. J. Eysenck-S. B. G. Eysenck and P. T. Costa-R. R. McCrae, we construct a knot model of a human mind, called a mind knot in terms of 2-bridge knots so that the knot type of a mind knot gives the personality. A mind relation is regarded as a link of mind knots, and we consider two and three mind relations modulo the self-releasability relations.

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1. Introduction

The purpose of this chapter is try to visualize a mind situation by constructing a knot model of a human mind (briefly, a mind knot) as an application of knot theory.

*This article is an improved version of the paper [10]

For several years ago (cf.[9]), the author considered that there is no contradiction by considering a mind as a knot, which is a main research object in knot theory, whose knot type is regarded as the personality and whose crossing change is regarded as a mind change, from the reason that several expressions on minds, such as a tame (straightforward) character, a twisted character, a string connecting human feelings, one's heartstring, an emotional entanglement on human relations, and being rooted in one's mind are represented in daily life by strings. Besides, the author knew by R. Rucker's book [13] that B. Stewart and P. G. Tait[1] appears to say in their book [17] that the soul exists as a knotted vortex ring in the aether, although "the aether hypothesis "is known as a wrong story and the author does not clearly understand what they mean. For our purpose, the following points are some difficulty points which we must overcome to construct our model (cf. [3]):

(1) (Birth-Time Mind Situation) By a genetic character inherited from Parents, we cannot always assume that the mind in birth time is untwisted.

(2)(Estimation on Changes of Environments) There are many sources of mind changes coming from changes of environments on

(i) Age
(ii) History
(iii) Non-standard event factors.

In our knot model of a mind, we count the conditions (1) and (2). Throughout this chapter, we mean by *knots* unoriented loops embedded in the 3-space R^3, and by *links*, disjoint unions of finitely many knots in R^3. The *types* of knots and links are understood as the equivalence classes of them under the Reidemeister moves in Figure 1. For general terms of knot theory, we refer to [8].

Concluding this introduction, we mention here some studies on applying topology to psychology. E. C. Zeeman's work on the topology of the brain and visual perception in [21] and the topological psychology by K. Lewin [11] which is an application of topological space are respectively pointed out by R. Fenn and by J. Simon with an implication by S. Kinoshita during the international conference " International Workshop on Knot Theory for Scientific Objects ". L. Rudolph reported to the author his works of several applications of topology to psychology which are published in [14, 15, 16] and his works (in preparation) on a rehabilitation of K. Lewin's topological psychology. Incidentally, the author also knew J. Valsiner's book [19] by Rudolph's report.

2. Constructing a knot model of a mind

To clarify our viewpoint of a mind, we set up the following mind hypothesis:

Mind Hypothesis 2.1.

[1]P. G. Tait is known as a pioneer of knot theory.

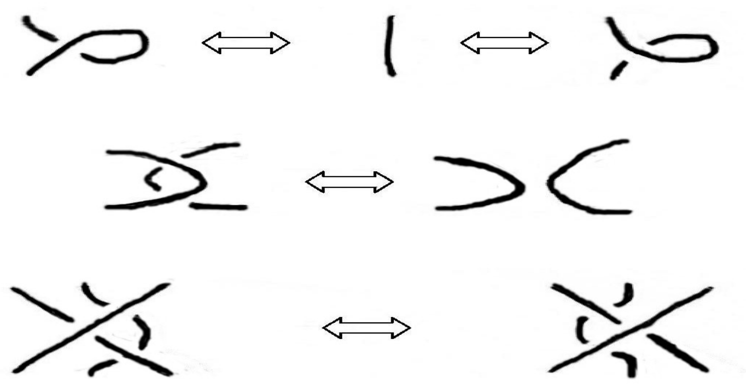

Figure 1: Reidemeister moves

(1) A *mind* is understood as a knot, so that an *untwisted mind* is a trivial knot and a *twisted mind* is a non-trivial knot.
(2) A personality is understood as the type of a knot, so that the *untwisted personality* is the knot type of a trivial knot and a *twisted personality* is the knot type of a non-trivial knot.
(3) A *mind change* is understood as a crossing change of a knot(see Figure 2).

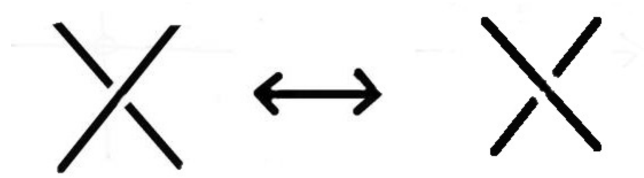

Figure 2: A crossing change

To construct a concrete knot model, we consider the basic factors of a mind by H. J. Eysenck [4, 5] and H. J. Eysenck-S. B. G. Eysenck [6, 7], which are described as follows:

(1) Introversion-Extroversion.

(2) Neuroticism.
(3) Psychoticism.

These basic factors are refined by P. T. Costa and R. R. McCrae [1] as Five-Factor Model (= Big-Five Model) described as follows:

(1) Introversion-Extroversion.
(2) Neuroticism.
(3_1) Openness to Experience.
(3_2) Agreeableness
(3_3) Conscientiousness

Our idea of constructing a knot model of a mind is to evaluate the degrees of the five-factors of the personality of a mind at the n-year-old as follows (See [20] for a concrete test) [2]:

Definition 2.2.

(1) The *introversion-extroversion degree* IE_n at n-year-old takes the value $\mathrm{IE}_n = -1, 0$ according to whether the mind is introverted or not.
(2) The *neuroticism degree* N_n at n-year-old takes the value $\mathrm{N}_n = -1, 0$ according to whether the mind is neurotic or not.
(3_1) The *openness to experience degree* O_n at n-year-old takes the value $\mathrm{O}_n = 0, -1$ according to whether the mind is open to the experience or not.
(3_2) The *agreeableness degree* A_n at n-year-old takes the value $\mathrm{A}_n = -1, 0$ according to whether the mind is disagreeable or not.
(3_3) The *conscientiousness degree* C_n at n-year-old takes the value $\mathrm{C}_n = -1, 0$ according to whether the mind is unconscientious or not.

We define the psychoticism degree at n-year-old from (3.1)-(3.3) as follows:

(3) The *psychoticism degree* OAC_n at n-year-old is the product

$$\mathrm{OAC}_n = \mathrm{O}_n \cdot \mathrm{A}_n \cdot \mathrm{C}_n = -1 \text{ or } 0.$$

We also need Father's and Mother's similar data IE_F, N_F, OAC_F and IE_M, N_M, OAC_M on their introversion-extroversion, neuroticism and psychoticism degrees at the baby's birth time, respectively. Namely, we have the following definition:

Definition 2.3.

(1) *Father's and Mother's introversion-extroversion degrees* IE_F and IE_M at the baby's birth time take the values $\mathrm{IE}_F = -1, 0$ and $\mathrm{IE}_M = -1, 0$ according to whether Father's mind and Mother's mind are introverted or not, respectively.
(2) *Father's and Mother's neuroticism degrees* N_F and N_M at the baby's birth time take the values $\mathrm{N}_F = -1, 0$ and $\mathrm{N}_M = -1, 0$ according to whether Father's mind and Mother's mind are neurotic or not, respectively.

[2] In this article, we use the simplest evaluation.

(3_1) *Father's and Mother's openness to experience degrees* O_F and O_M at the baby's birth time take the values $\mathrm{O}_F = 0, -1$ and $\mathrm{O}_M = 0, -1$ according to whether Father's mind and Mother's mind are open to the experience or not, respectively.
(3_2) *Father's and Mother's agreeableness degrees* A_F and A_M at the baby's birth time take the values $\mathrm{A}_F = -1, 0$ and $\mathrm{A}_M = -1, 0$ according to whether Father's mind and Mother's mind are disagreeable or not, respectively.
(3_3) *Father's and Mother's conscientiousness degrees* C_F and C_M at the baby's birth time take the values $\mathrm{C}_M = -1, 0$ and $\mathrm{C}_F = -1, 0$ according to whether Father's mind and Mother's mind are unconscientious or not, respectively.

We define Father's and Mother's psychoticism degrees at the baby's birth time from (3_1)-(3_3) as follows:

(3) *Father's and Mother's psychoticism degrees* Father's and Mother's psychoticism degrees OAC_F and OAC_M at the baby's birth time are respectively the products

$$\mathrm{OAC}_F = \mathrm{O}_F \cdot \mathrm{A}_F \cdot \mathrm{C}_F = -1 \ \text{or} \ 0 \quad \text{and} \ \mathrm{OAC}_M = \mathrm{O}_M \cdot \mathrm{A}_M \cdot \mathrm{C}_M = -1 \ \text{or} \ 0.$$

We define *Parents' introversion-extroversion, neuroticism and psychoticism degrees* IE_P, N_P and OAC_P by the identities

$$\mathrm{IE}_P = \ell_F \mathrm{IE}_F + \ell_M \mathrm{IE}_M,\ \mathrm{N}_P = m_F \mathrm{N}_F + m_M \mathrm{N}_M,\ \mathrm{OAC}_P = n_F \mathrm{OAC}_F + n_M \mathrm{OAC}_M,$$

respectively, for suitable non-negative integral constants ℓ_F, ℓ_M, m_F, m_M, n_F, n_M. In our argument here, we take $\ell_F = \ell_M = m_F = m_M = n_F = n_M = 1$, but the values shuold be chosen by estimating carefully a genetic character inherited from Parents (cf. [3]). The *total introversion-extroversion, neuroticism and psychoticism degrees* of a mind at n-year-old are respectively defined as follows:

$$\begin{aligned} \mathrm{IE}[n] &= \mathrm{IE}_P + \sum_{i=1}^{n} \mathrm{IE}_i, \\ \mathrm{N}[n] &= \mathrm{N}_P + \sum_{i=1}^{n} \mathrm{N}_i, \\ \mathrm{OAC}[n] &= \mathrm{OAC}_P + \sum_{i=1}^{n} \mathrm{OAC}_i. \end{aligned}$$

Let $a = \mathrm{IE}[n] + \mathrm{N}[n]$ and $b = \mathrm{OAC}[n]$. We have

$$-2n - 4 \leqq a \leqq 0 \quad \text{and} \quad -n - 2 \leqq b \leqq 0.$$

Our knot model of a mind at n-year-old is the knot $M^n = M^n(a, b)$ which is represented by the knot diagram with $|a| + 2|b|$ crossings, illustrated in Figure 3 and called the *n-year-old mind knot*. When the $(n-1)$-year-old mind knot diagram M^{n-1} is changed into a distinct n-year-old mind knot diagram M^n, we consider that some mind changes occur during the $(n-1)$-year-old and n-year-old. Thus, in our model of a mind knot, the total picture of a mind knot from the birth time to the n-year-old

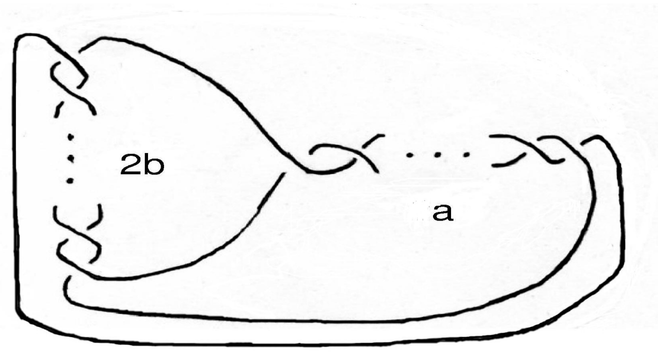

Figure 3: A mind knot $M^n(a,b)$

is considered as a cylinder properly immersed in the 4-dimensional space $R^3 \times [0,n]$. If we evaluate the degrees of the five-factors finer, then a finer mind knot model can be constructed. See § 5 later for some other mind knot models.

The following proposition is a consequence of the well-known classification of 2-bridge knots in knot theory (cf. [8, 18]).

Proposition 2.4. A mind knot $M^n(a,b)$ is untwisted if and only if $a=0$ or $b=0$. Two twisted mind knots $M^n(a,b)$, $M^{n'}(a',b')$ have the same personality if and only if $(a',b')=(a,b)$.

Proof. The knot $M^n(a,b)$ is a 2-bridge knot whose type is calculated as follows:

$$\frac{1}{2b+\cfrac{1}{a}} = \frac{a}{2ab+1}.$$

Thus, $M^n(a,b)$ is a trivial knot if and only if $2ab+1=\pm 1$, which is equivalent to $ab=0$ by observing that $ab \geqq 0$. Let $M^n(a,b)$ and $M^{n'}(a',b')$ be non-trivial knots. Then we note that a,b,a',b' are all negative integers. If they belong to the same knot type, then we have $2ab+1=\pm(2a'b'+1)$, so that $ab=a'b'>0$. Further, we have $a \equiv a' \pmod{2ab+1}$ or $aa' \equiv 1 \pmod{2ab+1}$. Suppose $a \neq a'$. Changing the roles of a,b and a',b' if necessary, we may assume that $|a|>|a'|>0$. If the first congruence occurs, then there is a positive integer k such that $|a|>|a-a'|=k(2ab+1)$ which is impossible. If the second congruence occurs, then we have

$$a(2b+a') \equiv 2ab+aa' \equiv -1+1=0 \pmod{2ab+1}.$$

Since a is coprime with $2ab+1$, we have $2b+a' \equiv 0 \pmod{2ab+1}$ and there is a positive integer k' such that

$$2|b|+|a| > |2b+a'| = k'(2ab+1),$$

which is impossible. Thus, we must have $a=a'$ and $b=b'$. □

For example, $M^n(-1,-1)$ is the negative trefoil knot and $M^n(-2,-1)$ is the figure-eight knot.

3. Further constructions on human mind models

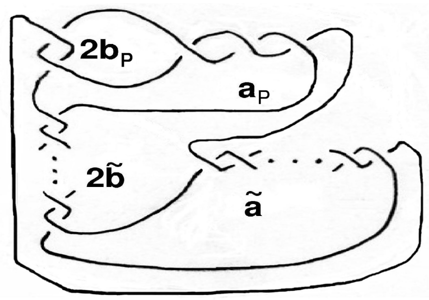

Figure 4: A mind knot $M^n(a_P, b_P, a, b)$

By taking $a_P = \mathrm{IE}_P + \mathrm{N}_P$, $b_P = \mathrm{OAC}_P$, $\tilde{a} = a - a_P$, and $\tilde{b} = b - b_P$, we have a different knot model $M^n(a_P, b_P, \tilde{a}, \tilde{b})$ of a mind, illustrated in Figure 4. If $b_P = 0$, then we have $M^n(a_P, b_, \tilde{a}, \tilde{b}) = M^n(0, 0, \tilde{a}, \tilde{b})$ where Proposition 2.4 can be used for the classification. However, if $b_P \neq 0$, then $M^n(a_P, b_P, a, b)$ is on a different behavior from $M^n(a, b)$ in general. We have also a further refined knot model by decomposing $\tilde{a}$ and $\tilde{b}$ into the numbers $a_i = \mathrm{IE}_i + \mathrm{N}_i$ and $b_i = \mathrm{OAC}_i$ $(i = 1, 2, \ldots, n)$, respectively, and by considering the knot $M^n(a_P, b_P, a_1, b_1, \ldots, a_n, b_n)$ illustrated in Figure 5. In our construction, we used the full-twist tangles of the numbers $a, a_P, \tilde{a}, a_i$ and $b, b_P, \tilde{b}, b_i$. If we use the half-twist tangles of these numbers instead of the full-twists, then our knot model changes into a link model which is a mind knot or a link of just two untwisted mind knots.

4. Self-releasability relations on mind links

When we consider a mind as a knot, it is an interesting research to consider a mind relation which we call a *mind link* as a link of mind knots. Every mind link is generated by a mind change on a mind knot component and a crossing change (which we also call a *mind change*) between a pair of mind knot components. We shall introduce a concept of self-releasability relations on mind links of $n(\geqq 2)$ mind knots.

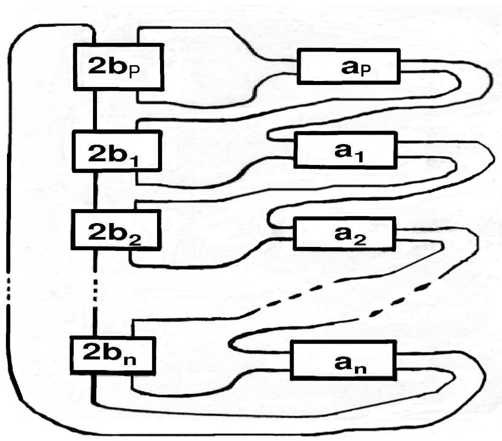

Figure 5: A mind knot $M^n(a_P, b_P, a_1, b_1, \ldots, a_n, b_n)$

Definition 4.1. Non-splittable mind links L and L' have the *same self-releasability relation* if there is a bijection τ from the mind knots of L onto the mind knots of L' such that for every mind sublink L_1 of L and every mind sublink L_2 of $L \backslash L_1$, we have

(1) the splittability between L_1 and L_2 coincides with the splittability between $\tau(L_1)$ and $\tau(L_2)$, and
(2) the splittability between L_1 and L_2 up to finitely many mind changes on every component of L_1 coincides with the splittability of $\tau(L_1)$ and $\tau(L_2)$ up to finitely many mind changes on every component of $\tau(L_1)$.

In Definition 4.1, we say that a mind sublink L_1 is *self-releasable* from L_2 if L_1 is made split from L_2 by finitely many mind changes on every component of L_1. The same self-releasability relation is an equivalence relation on mind links and the equivalence class of a non-splittable mind link is called a *self-releasability relation*. The classification of self-releasability relations for 2 mind knots is given as follows:

Proposition 4.2. The following cases (1)-(3) give the complete list of the self-releasability relations for 2 mind knots A and B.

(1) Neither mind knot component is self-releasable from the other. This relation is denoted by $A{-}B$.
(2) Both mind knots are self-releasable from each other. This relation is denoted by $A{\leftrightarrow}B$.
(3) One mind knot component, say A is self-releasable from the other mind knot-component B, but B is not self-releasable from A. This relation is denoted by $A{\rightarrow}B$. In this case, A is necessarily a twisted mind.

Proof. Let $A \cup B$ be a 2-component non-split link. For example, if the linking number $\mathrm{Link}(A, B) \neq 0$, then it satisfies (1), because A is not null-homotopic in the

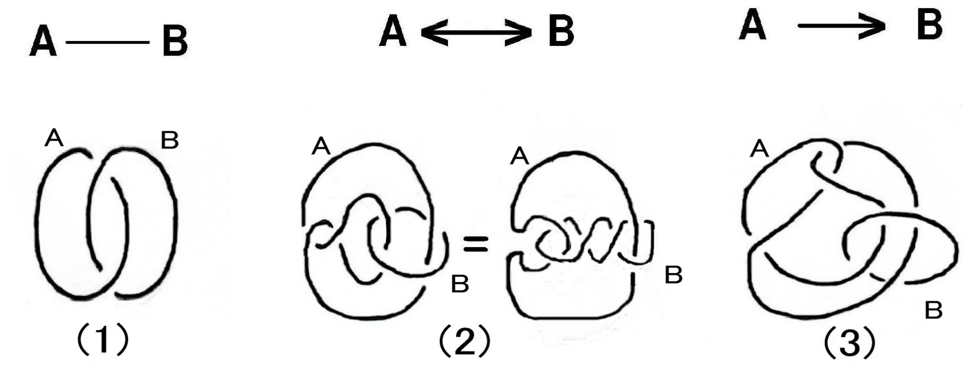

Figure 6: Self-releasability relations for two mind knots

space $R^3\backslash B$ and B is not null-homotopic in the space $R^3\backslash A$. If A and B are untwisted mind knots and the linking number $\mathrm{Link}(A, B) = 0$, then A and B are respectively null-homologous in the spaces $R^3\backslash B$ and $R^3\backslash A$ which are homotopy equivalent to S^1, so that A and B are null-homotopic in $R^3\backslash B$ and $R^3\backslash A$, respectively. Thus, this link in this case satisfies (2). If $\mathrm{Link}(A, B) = 0$ and B is an untwisted mind and not null-homotopic in $R^3\backslash A$ (in this case, A must be a twisted mind knot), then A is self-releasable from B, but B is not self-releasable from A, satisfying (3). The typical examples of the cases (1)-(3) are given in Figure 6. In (3) of Figure 6, we note that A is a trefoil knot and B is a trivial knot and represents the element $x^{-1}y$ in a group presentation $(x, y|xyx = yxy)$ of the fundamental group $\pi_1(R^3\backslash A)$ (see [2]). Since this group is non-abelian, we have $x^{-1}y \neq 1$ in $(x, y|xyx = yxy)$ and we see that B is not null-homotopic in $R^3\backslash A$. □

5. Self-releasability relations on three mind knots

We consider here self-releasability relations on three mind knots. The self-releasable relation of a non-splittable mind link of three mind knots A, B and C, consists of one triangle relation on A, B and C and a triplet of 1 : 2 relations on A, B, C consisting of

(1) $A{-}BC$ (meaning neither A nor $B \cup C$ is self-releasable from each other),
(2) $A{\leftrightarrow}BC$ (meaning A and $B \cup C$ are self-releasable from each other),
(3) $A{\rightarrow}BC$ (meaning A is self-releasable from $B \cap C$, but $B \cup C$ is not self-releasable from A), and
(4) $A{\leftarrow}BC$ (meaning $B \cup C$ is self-releasable from A, but A is not self-releasable from $B \cup C$), modulo permutations on A, B, C.

The list of triangle relations on three mind knots A, B and C together with one

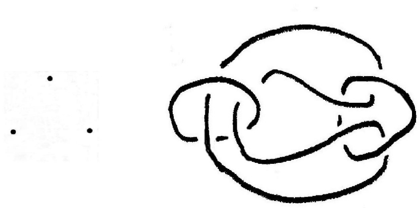

Figure 7: Borromean rings

mind link example for every triangle relation is given in Figure 8, Figure 9 and Figure 10. These figures are borrowed from Figure 6 and Figure 7 of [10] except the link diagrams of $3-8$, $3-20$ slightluy changed here and the incorrected schematic diagram of $3-11$ corrected here. We include the table of the triplets of $1:2$ relations of the examples to show how they are generated. The Borromean rings in Figure 7 is an interesting example such that every pair has no mind relation (thus, belonging to the triangle relation $3-1$), but the triplet of $1:2$ relations is $A-BC$, $B-CA$ and $C-AB$. This property is well-known by a stronger result on J. W. Milnor's link-homotopy classification in [12]. A complete classification for $n(\geqq 3)$ mind knots appears possible but complicated, and remains as an open problem.

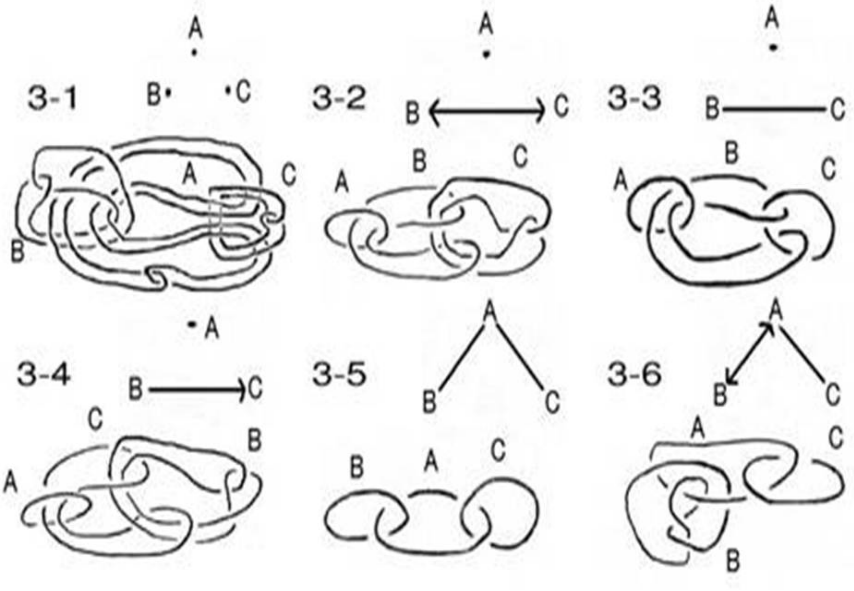

Figure 8: Self-releasability relations for three mind knots

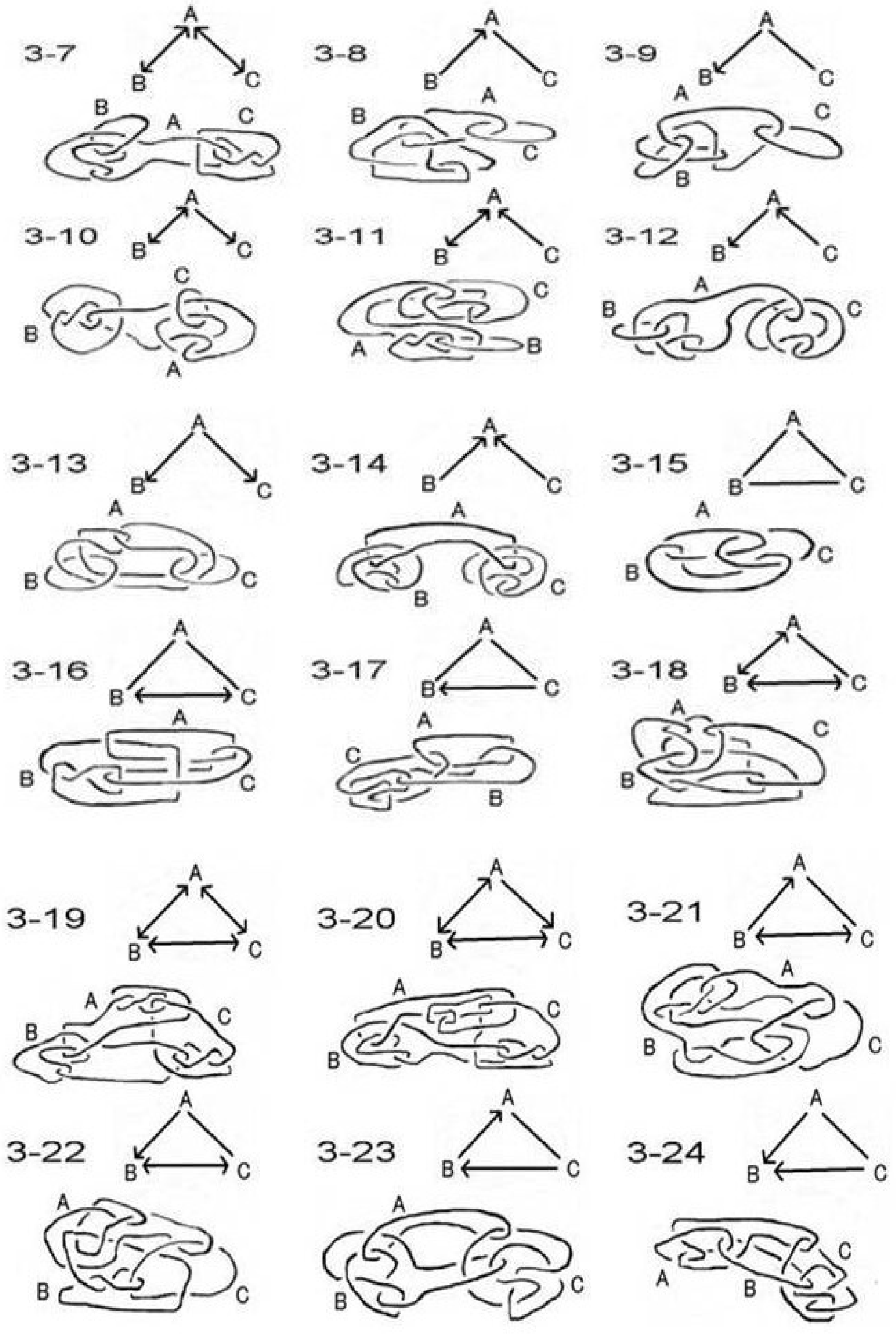

Figure 9: Self-releasability relations for three mind knots(continued)

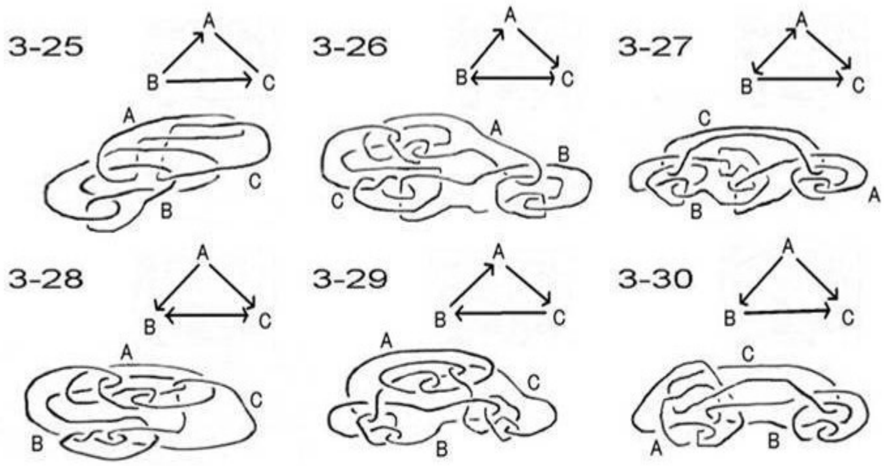

Figure 10: Self-releasability relations for three mind knots(continued)

The triplets of 1:2 relations of the mind links in Figures 8 and 9

Type onA, B, C	Relation onA, BC	Relation onB, CA	Relation onC, AB
$3-1$	$A\leftrightarrow BC$	$B\leftrightarrow CA$	$C\leftrightarrow AB$
$3-2$	$A\leftarrow BC$	$B\leftrightarrow CA$	$C\leftrightarrow AB$
$3-3$	$A\rightarrow BC$	$B-CA$	$C-AB$
$3-4$	$A\leftarrow BC$	$B\rightarrow CA$	$C\leftarrow AB$
$3-5$	$A-BC$	$B-CA$	$C-AB$
$3-6$	$A-BC$	$B\leftrightarrow CA$	$C-AB$
$3-7$	$A\leftrightarrow BC$	$B\leftrightarrow CA$	$C\leftrightarrow AB$
$3-8$	$A-BC$	$B\rightarrow CA$	$C-AB$
$3-9$	$A-BC$	$B\leftarrow CA$	$C-AB$
$3-10$	$A\rightarrow BC$	$B\leftrightarrow CA$	$C\leftarrow AB$
$3-11$	$A\leftarrow BC$	$B\leftrightarrow CA$	$C\rightarrow AB$
$3-12$	$A-BC$	$B\leftarrow CA$	$C\rightarrow AB$
$3-13$	$A\rightarrow BC$	$B\leftarrow CA$	$C\leftarrow AB$
$3-14$	$A\leftarrow BC$	$B\rightarrow CA$	$C\rightarrow AB$
$3-15$	$A-BC$	$B-CA$	$C-AB$
$3-16$	$A-BC$	$B-CA$	$C-AB$

Type onA, B, C	Relation onA, BC	Relation onB, CA	Relation onC, AB
$3-17$	$A{-}BC$	$B{-}CA$	$C{-}AB$
$3-18$	$A{-}BC$	$B{\leftrightarrow}CA$	$C{-}AB$
$3-19$	$A{\leftrightarrow}BC$	$B{\leftrightarrow}CA$	$C{\leftrightarrow}AB$
$3-20$	$A{\rightarrow}BC$	$B{\leftrightarrow}CA$	$C{\leftarrow}AB$
$3-21$	$A{-}BC$	$B{\rightarrow}CA$	$C{-}AB$
$3-22$	$A{-}BC$	$B{\leftarrow}CA$	$C{-}AB$
$3-23$	$A{-}BC$	$B{-}CA$	$C{-}AB$
$3-24$	$A{-}BC$	$B{\leftarrow}CA$	$C{-}AB$
$3-25$	$A{-}BC$	$B{\rightarrow}CA$	$C{-}AB$
$3-26$	$A{-}BC$	$B{\rightarrow}CA$	$C{\leftarrow}AB$
$3-27$	$A{\rightarrow}BC$	$B{\rightarrow}CA$	$C{\leftarrow}AB$
$3-28$	$A{\rightarrow}BC$	$B{\leftarrow}CA$	$C{\leftarrow}AB$
$3-29$	$A{-}BC$	$B{-}CA$	$C{-}AB$
$3-30$	$A{\rightarrow}BC$	$B{-}CA$	$C{\leftarrow}AB$

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