An abstract of the results of my research

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A knot is a circle S^1 embedded in a 3-sphere S^3. In knot theory, there exists the study of creating a prime knot table. In 1969, J. H. Conway made an enumeration of prime knots with up to eleven crossings by introducing the concept of a tangle and a basic polyhedron. A tangle is a pair consisting of a 3-ball B^3 and a (possibly disconnected) proper 1-submanifold t with $\partial t \neq \emptyset$, and a basic polyhedron is the 4-regular planar graph which has no bigon. We can obtain knots from basic polyhedra by substituting tangles for their vertices.

As a generalization of classical knot theory, we can consider other objects embed in S^3. A spatial graph is a graph embedded in S^3. If the graph consists of two vertices and three edges such that each edge joins the vertices, then it is called a θ-curve. There exist θ-curve invariants of ambient isotopy. For example, the constituent knots, the Alexander polynomial and the Yamada polynomial are well-known.

In his letter of 1989, R. A. Litherland announced a table of prime θ-curves with up to seven crossings, where the completeness of his table had not been proved. In master's thesis, I obtained all the prime θ-curves with up to seven crossings, which are the same table as Litherland's. First I constructed a prime basic θ-polyhedron to enumerate prime θ-curves. A θ-polyhedron is a connected planar graph embedded in 2-sphere, whose two vertices are 3-valent, and the others are 4-valent. Then our θ-polyhedron is different from Conway's polyhedron. Since I would like to make a prime θ-curve table, I omitted a non-prime θ-polyhedron. Then there exist twenty-four prime basic θ-polyhedra with up to seven 4-valent vertices. We can obtain θ-curves from prime basic θ-polyhedra by substituting tangles for their 4-valent vertices. Litherland classified these θ-curves by constituent knots and the Alexander polynomial. I classified them by the Yamada polynomial.

Moreover, I enumerated all the prime handcuff graphs with up to seven crossings. A handcuff graph is a spatial graph which consists of two loops and an edge jointing the vertices of each loop. There exist handcuff graph invariants of ambient isotopy. For example, the constituent link and the Yamada polynomial are well-known. I classified these handcuff graphs by using the Yamada polynomial.