Research plan

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1. As the final step of a review of basic facts on tilting theory in Grothendieck categories from the point of view of derived categories by using 1-tilting stalk complexes, I will study Grothendieck groups. Let $\mathcal{A}$ be a locally noetherian Grothendieck category. A Grothendieck group $K_0(\mathcal{A})$ is defined as follows :

$$K_0(\mathcal{A}) := \bigoplus_{[X]\in \operatorname{fg}\mathcal{A}/\cong} \mathbb{Z}[X]/I$$

where $\operatorname{fg}\mathcal{A}$ is the category of finitely generated objects of $\mathcal{A}$ and I is the subgroup generated by

$$\langle [X] - [Y] + [Z] | 0 \to X \to Y \to Z \to 0 \text{ exact}\rangle.$$

This definition is a natural generalization of Grothendieck group in the module categories. Let T be a 1-tilting stalk complex. Put $B = \operatorname{End}_{\mathcal{A}}(T)$. I will first give an alternative proof of $K_0(\mathcal{A}) \cong K_0(B)$.

2. The second purpose is to generalize my results on tilting theory in Grothendieck categories. Here, we suppose that the derived category $\mathcal{D}(\mathcal{A})$ of a Grothendieck category $\mathcal{A}$ is compactly generated. In this case, if $T \in \mathcal{A}$ is an n-tilting stalk complex, then it is an n-tilting object. But the converse is not true. A counter example due to D'Este is already known. I will observe under which conditions the converse is true. Based on this observation, I plan to prove basic facts on tilting theory in Grothendieck categories from the point of derived categories by using n-tilting stalk complexes.
3. The third purpose is to apply the results on tilting theory in Grothendieck categories to tilting sheaves. Namely, I will show that tilting sheaves are m-tilting stalk complexes for some m, and then plan to examine the differences between the definitions of tilting sheaves and of tilting stalk complexes.
4. I plan to study tilting modules. The study is related to the second purpose. Let A be a finite-dimensional algebra over an algebraically closed field and T a tilting module over A. Then the number of isomorphism classes of simple A-modules is equal to that of indecomposable direct summands of T. Now, there has been a problem whether this property may replace one of the conditions of a tilting module that there is an exact sequence: $0 \to A \to T_0 \to \cdots \to T_r \to 0$, where $T_0, \ldots, T_r \in \operatorname{add} T$. I will investigate under which conditions this is true. If T is a tilting module in the classical sense, this has been known to be true without conditions.