Research Plan

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1. Problems

I shall study the following problems:

(1) Evaluate the number of holomorphic sections of holomorphic families of Riemann surfaces.

(2) Show the global non-triviality of the above triple $(\mathcal{M}, \pi, R)$.

2. Plans

(1) First, I shall have the number of holomorphic sections of concrete holomorphic families. Next, I will conjecture the number of holomorphic sections of general ones. It is known that every holomorphic family and its holomorphic sections are determined by the monodromy of the family （Imayoshi ＆ Shiga's Rigidity Theorem）. Thus, if I decide the monodromy, then I can estimate the number of holomorphic sections. And by use of 2,3 dimensional hyperbolic geometries and the theory of Kleinian groups, I can determine the monodromy.

(2) Kodaira surface, which was a holomorhic family constructed by Kodaira, was showed to be locally non-trivial. And $(\mathcal{M}, \pi, R)$ was also showed to be locally non-trivial. Then, is $(\mathcal{M}, \pi, R)$ globally non-trivial? We give a defining equation of it. So first, I shall study this problem with the equation.

At the same time, given two sets of six distinct points on the Riemann sphere, I study when the two sets are mapped to each other by a Möbius transformation. Since each fiber of $(\mathcal{M}, \pi, R)$ is a closed Riemann surface of genus two, it is represented as a two-sheeted branched covering surface of the Riemann sphere branched over six points. It is well known two fibers S and S' are biholomorphically equivalent if and only if there is a Möbius transformation which takes the set of branch points of S to the set of branch points of S'. By use of the theory of configuration spaces, we can study the problem.