Summary of my research

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A μ component link L is μ circles embedded in 3-Euclidean space $\mathbb{R}^3$(or 3-sphere S^3). Particularly, a circle embedded in $\mathbb{R}^3(\mu = 1)$ is called a knot.

Two links L and L' are equivalent if there is a orientation preserving homeomorphism $\varphi : (\mathbb{R}^3, L) \longrightarrow (\mathbb{R}^3, L')$. There are various kinds of link invariants that is the mathematical value which take the same value for the equivalent links.

I studied Alexander polynomial that is a well-known link invariant. Especially, I studied the trapezoidal conjecture for Alexander polynomials of alternating links. The trapezoidal conjecture is announced by R. H. Fox in 1961. The claim of the trapezoidal conjecture is as follows. Let L be an alternating link and $\Delta_L(t)$ be the Alexander polynomial of L. Then $\Delta_L(-t)$ will be a trapezoidal polynomial. Where a polynomial $f(t) = \sum_{i=0}^{m} c_j t^j (c_j \in \mathbb{N})$ is trapezoidal if which meets following three properties.

- $c_i = c_{m-i}$ for $i = 0, 1, \ldots, m$.
- $0 < c_0 \leq c_1 \leq \cdots \leq c_{[\frac{m}{2}]}$.
- If exist an integer j such that $c_j = c_{j+1}$, then $c_k = c_j$ for $k = j, \ldots, [\frac{m}{2}]$.

In 1979, R. I. Hartley proved that the trapezoidal conjecture is true for two-bridge links. In 1985, K. Murasugi proved that the trapezoidal conjecture is true for alternating algebraic links.

By applying the matrix-tree theorem to $\Delta(-t) = det(-tV - V^T)$, I shown that the Alexander polynomials of special alternating links are interpreted a sum of t^ℓ-weighted monocyclic states.

In 2005, A. Stoimenow proved that any prime special alternating knot diagram is obtained by applying some $\overline{t'_2}$-moves to 11 knot diagrams(5_1, 7_5, 8_{15}, 9_{23}, 9_{38}, 10_{101}, 10_{120}, 11_{123}, 11_{329}, 12_{1097}, 13_{4233}).

By using the method of monocyclic state-sum, I studied the affect of applying a $\overline{t'_2}$-move to special alternating link diagram. Consequently, I proved that the trapezoidal conjecture is true for special alternating knots of genus one and two.