A research project

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A surface-link is a 2-dimensional closed submanifold of R^4 (or S^4). A surface-link can be represented by some 4-valent plane graph having two kinds of vertices which is called a ch-diagram. K. Yoshikawa made a table of all surface-links which can be represented by ch-diagrams with vertices less than or equal to ten. It was indicated by S. Kamada that any immersed closed surfaces in R^4 (or S^4) which has only transversal double points can be represented by some 4-valent plane graph having three kinds of vertices, which is called a ch-diagram with double points. I made a table of such surfaces which can be represented by ch-diagrams with double points the number of whose vertices is less than or equal to five. I want to enumerate all such diagrams with vertices less than or equal to six.

In order to make the enumeration of ch-diagrams more efficient, Yoshikawa defined ch-graphs which are obtained from ch-diagrams in a similar way to construct graphs for 1-dimensional link diagrams. He defined local moves on ch-graphs which correspond to that on ch-diagrams, using these moves, reduced the number of ch-graphs which should be checked for enumeration, and made such a table successfully.

By analogy with his method, I define ch-graphs with double points for ch-diagrams with double points to enumerate immersed closed surfaces. It is however difficult without any improvement to reduce the number of ch-graphs with double points which should be checked and it costs very much time to enumerate all ch-diagrams with double points the number of whose vertices is less than or equal to six. Therefore by studying precisely the properties of ch-diagrams with double points and the ch-graphs, I want to contribute towards efficient enumeration of such diagrams to make the table mentioned above.