An abstract of the results of my research

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A surface-link F is a 2-dimensional closed submanifold of Euclidean 4-space R^4 (or 4-sphere S^4). If F is connected, then it is called a surface-knot. A ch-diagram was introduced by S. J. Lomonaco, Jr. and K. Yoshikawa as a way to represent a surface-knot. A ch-diagram is a 4-valent plane graph which has two kinds of vertices. Yoshikawa defined eight local moves Ω_1–Ω_8 on ch-diagrams. These moves preserve the types (up to ambient isotopies in R^4) of surface-knots represented by such diagrams. I defined three more local moves Ω_9–Ω_{11} on ch-diagrams (which do not preserve the types of surface-knots in general), and gave an algorithm to transform any ch-diagram into a trivial one by these eleven moves. As a corollary, it is clear that we can obtain the genus of the surface-knot represented by a ch-diagram from the numbers of the three moves Ω_9–Ω_{11} in this transformation. This result is published in the paper entitled "An unknotting sequence for surface-knots represented by ch-diagrams and their genera" in Kobe Journal of Mathematics Vol. 18 (2001) 163–180.

Yoshikawa made a table of all surface-links which can be represented by ch-diagrams with vertices less than or equal to ten. It was indicated by S. Kamada that any immersed closed surface (which has only transversal double points) can be represented by some 4-valent plane graph having three kinds of vertices, which is called a ch-diagram with double points. I applied Yoshikawa's method to enumerate such immersed closed surfaces. Eight local moves Ω_1–Ω_8 is also valid for ch-diagrams with double points; if two ch-diagrams with double points representing immersed closed surfaces are related by a finite sequence of these moves, then the two surfaces are ambient isotopic in R^4. I defined some more local moves on ch-diagrams with double points (which preserve the types of immersed closed surfaces). I enumerated all immersed closed surfaces which can be represented by ch-diagrams with double points the number of whose vertices is less than or equal to five, and I am just doing it for ones with vertices less than or equal to six.