2.1 Reseaching plan

1. On a necessary and sufficient condition on freeness of the equivariant cohomology $H_T^*(\mathscr{G})$ of a GKM graph $\mathscr{G}$ as $H^*(BT)$-module.
   $H_T^*(\mathscr{G})$ is isomorphic to $\mathbf{k}[P(\mathscr{G})]$ by the theorem above. Therefore, our problem is replaced by a problem to find a necessary and sufficient condition of Cohen-Macaulayness of $\mathbf{k}[P]$ of simplicial poset P. This is a generalization of Reisner's theorem for a Stanley-Reisner ring of a simplicial complex. We may state this condition in terms of GKM graphs. For this purpose, we consider equivariant cohomology of blow-up of a GKM graph (defined as an analogue of blow-up of manifolds) and a sub GKM graph (an analogue of submanifolds). On the other hand, we can construct T-spaces over simplicial posets, which generalize Davis-Januszkiewics construction of T-spaces over simplicial complexes. Equivariant cohomology of the T-space is expected to be isomorophic to the face ring of the underlying simplicial poset by the result of Davis and Januszkiewics. From this point of view, we will be able to study equivariant cohomology $H_T^*(\mathscr{G})$ of GKM graphs using results on topology of spaces.

2. On K-theory of GKM graphs.
   Equivariant K-groups of GKM manifolds can be computed from associated GKM graphs similarly to the case of equivariant cohomology, following Knutson and Rosu. Therefore, we can define equivariant K-groups of GKM graphs, and develop equivariant K-theory. Moreover, we may define general cohomology theory on GKM graphs.

3. On genera of GKM graphs.
   Genera are defined as cobordism invariant of oriented compact manifolds. We may apply this concept to GKM graphs. In case of GKM manifolds, we can describe genera by the result of localization theorem. Therefore, we may define a genus of GKM graphs by analogue of this local formula. Moreover, we may introduce a notion of cobordism on GKM graphs (correspond to cobordism of manifolds), and investigate whether genera of GKM graphs defined above are cobordism invariant.