1.1 Abstract of researching results

Goresky, Kottwitz and MacPherson have shown that, when a compact Lie group G acts on a space X which satisfies a certain condition with finitely many isolated fixed points, the equivariant cohomology of X can be computed from the structure of the zero and one dimentional orbits of the action of a maximal torus of G.

The structure of these orbits can be represented by a finite simple d-valent graph with a labeling on oriented edges of the graph.

Based on the fact mentioned above, we consider an action of a compact torus T on a smooth manifold M with fixed point set M^T finite, which satisfies a certain condition about tangential representations of the T-action (we call this T-manifold a GKM manifold). Studying the pair (Γ, α), Γ being a finite simple d-valent graph and α a labeling (by degree 2 cohomology $H^2(BT)$ of classifying space BT of T) on oriented edges of the graph, we can get necessary conditions (about the labeling).

For a pair (Γ, α) which satisfies these conditions (we call GKM graph), we define the equivariant cohomology of (Γ, α), motivated by the results of Goresky-Kottwitz-MacPherson.

I studied properties of GKM graphs associated with actions of n-dimentional torus on $2n$-dimentional manifolds and the ring structure of equivariant cohomology of these GKM graphs.

Let $\mathscr{G} = (\Gamma, \alpha)$ be GKM graph. For all $k \leq n$, we can define the k-face F of $\mathscr{G}$ as a connected k-valent subgraph of Γ. Each k-face is a GKM graph with the labeling obtained by restriction of α to the subgraph. An intersection of faces is a union of faces. Moreover, we can define the equivariant cohomology class $\tau_F \in H_T^{2(n-k)}(\mathscr{G})$, called Thom class of F, to each face F.

The set $P(\mathscr{G})$ of faces of GKM graph $\mathscr{G}$ is a simplicial poset with respect to the reverse inclusion relation. We can define the face ring $\mathbf{k}[P]$ for a simplicial poset P, which reduces to Stanley-Reisner ring when P is a simplicial complex. It is the quotient of free module, generated by elements of P, by the ideal defined from the face structure of P. Now, we have a relation between the equivariant cohomology $H_T^*(\mathscr{G})$ of a GKM graph $\mathscr{G}$ and the face ring of the poset $P(\mathscr{G})$.

Theorem. $H_T^*(\mathscr{G})$ is isomorphic to $\mathbf{k}[P(\mathscr{G})]$ as ring, by correspondence Thom classes of faces to generators.