1.1 Abstract of researching results

Goresky, Kottwitz and MacPherson have shown that, when a T-space M with isolated fixed points is equivariantly formal, the image of the inclusion map $i : H_T^*(M) \to H_T^*(M^T)$ can be computed from the combinatorial structure of the zero and one dimentional orbits of the action of the torus T. Motivated by this result, Guillemin and Zara introduced the notion of graphs $\mathscr{G}$ with an axial function, which we call GKM graphs. They defined equivariant cohomology $H_T^*(\mathscr{G})$ of $\mathscr{G}$, and studied the relation of $H_T^*(M)$ and $H_T^*(\mathscr{G}_M)$, where $\mathscr{G}_M$ is the GKM graph associated with a GKM manifold M.

Based on the fact mentioned above, we consider an action of a torus T on a smooth manifold M with isolated fixed points, which satisfies a certain condition about tangential representations of the T-action (we call this T-manifold a GKM manifold). We can get a GKM graph $\mathscr{G}_M = (\Gamma, \alpha)$, Γ being a finite n-valent graph and $\alpha : E(\Gamma) \to \mathfrak{t}^* \cong H^2(BT)$ an attaching map, from the combinatorial structure of the zero and one dimentional orbits of the GKM manifold M.

I studied properties of GKM graphs and the ring structure of equivariant cohomology of GKM graphs.

For $k \leq n$, we can define the k-face F of $\mathscr{G}$ as a connected k-valent subgraph of Γ which is invariant under the connection of $\mathscr{G}$. (The connection of $\mathscr{G}$ is an additional structure of $\mathscr{G}$.) Moreover, we can define the Thom class $\tau_F \in H_T^{2(n-k)}(\mathscr{G})$ to each face F.

The set $P(\mathscr{G})$ of faces of a GKM graph $\mathscr{G}$ is a simplicial poset with respect to the reverse inclusion relation. A simplicial poset P is a "generalized" simplicial complex, which allows that an intersection of simplices is a *union* of simplices. We can define the face ring $\mathbf{k}[P]$ for a simplicial poset P, which is introduced by Stanley in combinatorics. It is the quotient of a polynomial ring generated by elements of P, by the ideal defined from the face structure of P.

Theorem. $H_T^*(\mathscr{G})$ is isomorphic to $\mathbf{k}[P(\mathscr{G})]$ as a ring, by correspondence an Thom class of each face to the generator associated to the face.

We can generalize a construction, given by Davis and Januszkiewicz, of a T-space over a dual space of a simplicial complex. For a simplicial poset P, We construct T-space $M(P)$ over a dual space of P, and there is the isomorphism $H_T^*(M(P)) \cong \mathbb{Z}[P]$.

From the fact about the blow-up of T-spaces and GKM graphs, we can see the following.

Theorem. Let $\widetilde{\mathscr{G}}$ be the blow-up of $\mathscr{G}$. $H_T^*(\mathscr{G})$ is a free module if and only if $H_T^*(\widetilde{\mathscr{G}})$ is a free module.