An abstract of the results of my research

Hiromasa Moriuchi

A knot is a circle S^1 embedded in a 3-sphere S^3. In knot theory, there exists the study of creating a prime knot table. In 1969, J. H. Conway made an enumeration of prime knots with up to eleven crossings by introducing the concept of a tangle and a basic polyhedron. A tangle is a pair consisting of a 3-ball B^3 and a (possibly disconnected) proper 1-submanifold t with $\partial t \neq \emptyset$, and a basic polyhedron is the 4-regular planar graph which has no bigon. We can obtain knots from basic polyhedra by substituting tangles for their vertices.

As a generalization of classical knot theory, we can consider other objects embed in S^3. A spatial graph is a graph embedded in S^3. If the graph consists of two vertices and three edges such that each edge joins the vertices, then it is called a θ-curve. There exist θ-curve invariants of ambient isotopy. For example, the constituent knots, the Alexander polynomial and the Yamada polynomial are well-known.

In his letter of 1989, R. A. Litherland announced a table of prime θ-curves with up to seven crossings, where the completeness of his table has not been proved. In master's thesis, I obtained all the prime θ-curves with up to seven crossings, which are the same table as Litherland's. First I constructed a prime basic θ-polyhedron to enumerate prime θ-curves. A θ-polyhedron is a connected planar graph embedded in 2-sphere, whose two vertices are 3-valent, and the others are 4-valent. Then our θ-polyhedron is different from Conway's polyhedron. Since I would like to make a prime θ-curve table, I omitted a non-prime θ-polyhedron. Then there exist twenty-four prime basic θ-polyhedra with up to seven 4-valent vertices. We can obtain θ-curves from prime basic θ-polyhedra by substituting tangles for their 4-valent vertices. Litherland classified these θ-curves by constituent knots and the Alexander polynomial. I classified them by the Yamada polynomial.