Summary of research

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The spin structure on principal $SO(n)$ bundle P is equivalence class of a $2-$fold covering which is non -trivial on each fibres. Let M be an oriented n-dimensional manifold, $F(M)$ be its frame bundle. $F(M)$ is principal $SO(n)$ bundle . A spin structure on M is the spin structure of $F(M)$. The correspondence between the spin structures on M and the mod 2 cohomology classes is not canonical. But, D. Johnson gave a canonical correspondence between the spin structures on the surface and the quadratic forms on the mod 2 homology group. Since the quadratic forms have an algebraic invariant, this correspondence defines invariant of the spin structures. This invariant was originally defined by M. Atiyah by using mod 2 index of Dirac operator and KO-theory. Therefore, it can be said that the method by Johnson is a topological one.

Let Σ_g be a closed oriented surface of genus g and Γ_g be its mapping class group , that is, the group of isotopy classes of orientation preserving diffeomorphisms of Σ_g. Γ_g is generated by finitely many Dehn twist around simple closed curves in Σ_g. S. Humphries showed that the minimal number of twist generators for Γ_g was $2g+1$. To prove that fewer than $2g+1$ do not suffice, he used certain graphs concerning the intersection of simple closed curves.

I got quadratic forms on homology group by using Humphries graphs. So I have been researching relationships between the spin structures and the mapping class group. In master's thesis, I reproved the result of Humphries by using the action of the mapping class group to an affine space consisting of the spin structures. I research relationships between the mapping class group and K-theory.