Summary of my research

Kenichi Masui

We have studied Denjoy system. Here, I will describe Denjoy system.

An orientation-preserving homeomorphism on the circle which has no periodic point has a irrational number as rotation number. In addition, if it is not topologically conjugate to irrational rotation, it is called Denjoy homeomorphism. (For more details of Denjoy homeomorphism, for example, refer to (Kouichi Yano, *Dynamical System 2* (in Japanese)). Denjoy system is defined as unique minimal subsystem of Denjoy homeomorphism, which is a Cantor minimal system. Then the set of the multiple point of factor map is at most countable orbits ([Poincare]). We call the number of these orbits double orbit number, and call this set double point set Besides the family of Denjoy systems whose elements are (orientation-preserving) topologically conjugate each other is determined from rotation number and the configuration of double point set (up to rotation) ([Markley]).

With respect to Cantor minimal system, R. H. Herman, I. F. Putnam and C. F. Skau have shown that there exists an adic model (adding machine defined on the infinite path space of Bratteli diagram) which is topologically conjugate to it.

We constructed concretely an adic model of Denjoy system whose double orbit number is finite. We use the continued fraction expansion of rotation number and associated Ostrowski type expansion for this construction.

Moreover for Cantor minimal systems, there exists an invariant value called dimension group, and it is known that the orbit structure of Cantor minimal system is intimately related to it ([T. Giordano, I. F. Putnam, C. F. Skau]). For the dimension group of Denjoy system, I. Putnam, K. Schmidt and C. Skau determined almost all.

Using this adic model, we could determine dimension group of Denjoy system of finite double orbit number, which contain the case they couldn't determine.

Moreover, transforming this a little, we get an adding machine define on the infinite path space of constant rank Bratteli diagram. This induces natural substitution system. In fact, this generates a (double orbit number +1) letters two-sided infinite sequence, and the subshift on the orbit closure of this is topologically conjugate to original Denjoy system. Especially, when double orbit number is 1, this infinite sequence is known as Sturmian sequence. In this meaning, we get a substitution system which generates a kind of the expansion of Sturmian sequence. This sequences is a coding sequence by disrupting the circle (double orbit number +1) arcs.