

Summary of my research

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Papers are referred by numbers in “List of my papers”

Due to the Hyperbolic Uniformization Theorem for Haken manifolds by Thurston the complements of almost all knots and links admit complete hyperbolic structures of finite volume. (We call such a manifold a cusped hyperbolic manifold.) Epstein and Penner defined an ideal polyhedral decomposition of a cusped hyperbolic manifold which is determined canonically from the hyperbolic structure. The decomposition is called the canonical decomposition of a manifold. I study to reveal the relationship between the hyperbolic and the topological structures of a cusped hyperbolic 3-manifold by means of canonical decomposition. My study so far is classified roughly into three lines; the first is a research on the relationship between Dehn surgeries and canonical decompositions, the second is the one between Heegaard splittings and hyperbolic structures and the third is the study of infinite volume hyperbolic manifolds from the view point of combinatorial structures.

Dehn surgeries and canonical decompositions: The canonical decomposition of a manifold which is obtained from a hyperbolic manifold with at least 2 cusps by Dehn filling on some of the ends can be characterized by using the combinatorial structure of the Ford domain of the cyclic Kleinian group corresponding to the core curve of the filling, for almost every surgery slope $((1), (2), (5), (11))$.

Heegaard splittings and canonical decompositions: (with M. Sakuma, M. Wada and Y. Yamashita): The characterization of the combinatorial structure of Ford domains of Kleinian (once) punctured torus groups due to T. Jorgensen by 1970s in an unfinished manuscript can be generalized to those of “Ford domains” of certain hyperbolic cone-manifolds, and the hyperbolic structures of all hyperbolic 2-bridge knots and links are concretely constructed. The canonical decomposition determined in such a way is seen to coincide with the topological ideal tetrahedral decompositions given by Sakuma and Weeks as candidates to the canonical decompositions $((4), (7), (8), (13))$. In our method, a continuous family of hyperbolic structures, each of which has cone singularities along the unknotting tunnels of the two-bridge knot under consideration called the upper and the lower tunnels, is constructed. The study in this line can be regarded as the first step to the construction of “hyperbolic Heegaard splitting theory”.

Hyperbolic manifolds of infinite volume: Jorgensen’s theory is seen to be able to be applied to the study of the Ford domains of geometrically infinite boundary groups. As a result, I could determine the canonical decomposition of any hyperbolic once-punctured torus bundle. (See (16) for an outline; details is in preparation.) Lackenby have independently determined the canonical decompositions of once-punctured torus bundles by using a combinatorial argument of polyhedral decompositions. This proves a beautiful correspondence of hyperbolic geometry and topology with canonical decompositions in between.

Recent results in the theory of Kleinian groups tells that the quotient of the hyperbolic 3-space by any Kleinian group is homeomorphic to the interior of a compact 3-manifold, and it is isometrically characterized by the end invariant. For punctured torus groups, the “side parameter” is also defined by using the combinatorial structure of Ford domain, and it takes values in the same space, which is the product of two copies of the Thurston compactified Teichmüller space, as end invariant. It is shown that the Weil-Petersson distance between the two invariants are bounded above by a universal constant. (See (21) for an outline; details is in preparation.)

From the point of view of Kleinian groups, it is essential to show the “Fixed Point Theorem” for the skinning map on the space of hyperbolic structures on the manifold obtained from original one by cutting along an essential surface, in order to construct the hyperbolic structure of non-fibered manifold. I am studying the geometrically finite hyperbolic structures on the manifolds obtained from the product of the once-punctured torus and the interval by Dehn surgery along an essential loop on a level surface, and have constructed numerically a certain path from a simple hyperbolic structure to the fixed point of the skinning map on which the combinatorial structure of the Ford domain is described.