

## Research Plan

### **(1) The cohomology rings of regular Hessenberg varieties and relations with other research areas**

We studied the cohomology rings of regular nilpotent Hessenberg varieties and regular semisimple Hessenberg varieties, and the associated relation with other research areas. There is a notion of regular Hessenberg varieties which are connecting between regular nilpotent Hessenberg varieties and regular semisimple Hessenberg varieties.

Recently, we obtained the result that the cohomology ring of a “minimal” regular Hessenberg variety is isomorphic to the cohomology ring of the (projective) toric orbifold associated with the polytope obtained by cutting the permutohedron along certain hyperplanes (in preparation). I continue to research the cohomology rings of regular Hessenberg varieties and the associated relation with other research areas.

### **(2) Stanley-Stembridge conjecture**

Stanley-Stembridge conjecture is a positivity problem for chromatic symmetric functions in graph theory. Surprisingly, it is known that the permutation group action (Tymoczko’s dot action) on the cohomology of a regular semisimple Hessenberg variety is equivalent to the chromatic symmetric function for certain graph. From this connection, Stanley-Stembridge conjecture is reduced to study the representations of the symmetric group on the cohomology of regular semisimple Hessenberg varieties. We studied the cohomology rings of special regular semisimple Hessenberg varieties, and using them, we observed Stanley-Stembridge conjecture is true for the special cases. From this way, I will try to solve Stanley-Stembridge conjecture.

### **(3) Harada-Tymoczko conjecture**

We obtained an algorithm for deriving a basis for the set of linear relations satisfied by the images of the Schubert classes in the cohomology of a regular nilpotent Hessenberg variety. Then, we can ask a natural question: Which Schubert classes in the cohomology of a regular nilpotent Hessenberg variety form a basis of the the cohomology of the regular nilpotent Hessenberg variety? Since Harada and Tymoczko conjectured the candidates, I will try this problem.

### **(4) Schubert calculus on regular nilpotent Hessenberg varieties**

I will study Schubert calculus on regular nilpotent Hessenberg varieties. When a regular nilpotent Hessenberg variety is the Peterson variety, we could describe combinatorially the structure constants for the intersections between Schubert varieties and the Peterson variety (in preparation). I continue to research how the structure constants in regular nilpotent Hessenberg varieties can be described combinatorially.