Research Results

I am interested in geometry and topology with torus group action, and relations to other research areas associated with them. The followings are main research results.

(1) The cohomology rings of regular nilpotent Hessenberg varieties and regular semisimple Hessenberg varieties (1.[5] in the List of papers)

The regular nilpotent Hessenberg varieties are the family of subvarieties of the flag variety connecting between the Peterson variety and the flag variety. On the other hand, the regular semisimple Hessenberg varieties are the family of subvarieties of the flag variety connecting between the permutohedral variety and the flag variety. Here, the Peterson variety is related with the quantum cohomology of (partial) flag varieties and the permutohedral variety is the projective toric variety associated with the permutohedron.

We first gave an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in type A. Then, using this explicit presentation, we proved that the cohomology ring of a regular nilpotent Hessenberg variety is isomorphic to the invariants under the symmetric group action of the cohomology ring of the regular semisimple Hessenberg variety where the symmetric group action is called Tymoczko's dot action. This is joint work with Hiraku Abe, Megumi Harada, and Mikiya Masuda.

(2) Hessenberg varieties and hyperplane arrangements (1.[3] and 2.[2] in the List of papers)

We obtained the surprising result that the cohomology rings of regular nilpotent Hessenberg varieties in all Lie types can be described in terms of hyperplane arrangements. From this result, we proved that the isomorphism of cohomology rings between regular nilpotent Hessenberg varieties and regular semisimple Hessenberg varieties explained in (1) is true for any Lie types. We also proved Peterson's announcement and Sommers-Tymoczko conjecture. Moreover, this surprising connection represents a significant step towards the solution for the problem of giving explicit presentations of the cohomology rings of regular nilpotent Hessenberg varieties. In fact, we gave explicit presentations for types B, C, G. These are joint work with Takuro Abe, Mikiya Masuda, Satoshi Murai, and Takashi Sato. Then, we obtained explicit presentations of regular nilpotent Hessenberg varieties for any Lie types. This is joint work with Makoto Enokizono, Takahiro Nagaoka, and Akiyoshi Tsuchiya.

(3) An additive basis for the cohomorogy ring of regular nilpotent Hessenberg varieties (1.[1] and 2.[1] in the List of papers)

We constructed an additive basis for the cohomology ring of a regular nilpotent Hessenberg variety, which is extending the Poincaré duals of all smaller regular nilpotent Hessenberg varieties in the given regular nilpotent Hessenberg variety. This is joint work with Makoto Enokizono, Takahiro Nagaoka, and Akiyoshi Tsuchiya.

After that, we constructed an filtration on the cohomology rings of regular nilpotent Hessenberg varieties, and obtained a monomial basis for the cohomology ring of a regular nilpotent Hessenberg variety, which is different than the additive basis above. When a regular nilpotent Hessenberg variety is the flag variety, the monomial basis is appeared in Schubert polynomials. Moreover, we gave an algorithm for deriving a basis for the set of linear relations satisfied by the images of the Schubert classes in the cohomology of a regular nilpotent Hessenberg variety.