Research Plan

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1 Enumeration and classification of ribbon 2-knots

We continue to enumerate and classify ribbon 2-knots.

(i) We enumerate and classify the ribbon 2-knots of 1-fusion with length up to 7.

(ii) We enumerate and classify the ribbon 2-knots with ribbon crossing number 5. In this family there exist ribbon 2-knots of 2-fusion.

2 Partial order for ribbon 2-knots

For two classical knots J and K, we write $J \geq K$ if there exists a surjective group homomorphism from $G(J)$ onto $G(K)$, where $G(K)$ is the knot group of K, the fundamental group of the complement of K. This relation satisfies the condition of a partial order on the set of prime knots. Kitano and Suzuki studied this partial order among prime classical knots, where they made use of the twisted Alexander polynomial. We would like to consider an analogous relation among ribbon 2-knots.

3 Fiberedness of a ribbon 2-knot

There have been a great progress in the study of fiberedness for classical knots using the twisted Alexander polynomial. For a ribbon fibered 2-knot of 1-fusion we gave a certain condition of the twisted Alexander polynomial and determined fibered ribbon 2-knots with ribbon crossing number up to 4. We would like to generalize such a condition for a general ribbon 2-knot.

4 Classification of ribbon knots with symmetric union presentations

The symmetric union introduced by Kinoshita and Terasaka and its generalization is a well-known method to construct a ribbon knot. Lamm gives many examples of symmetric unions and he asks whether every ribbon knot is a symmetric union representation. Besides this, Eisermann and Lamm introduced a notion of symmetric equivalence among symmetric union diagrams. We would like to consider a classification of knots presented as symmetric union. We can find several examples sharing the same polynomial invariants such as the Alexander, Conway, Jones, HOMFLYPT, Q, or Kauffman polynomials. For a particular family of knots we have difficulty in classifying them.