

Research plan (Mikiya Masuda) January, 2022

I would like to regard toric topology widely as mathematics related to torus actions and extend and deepen this area. Currently, I am working on the following three projects.

(1) The permutohedral variety is a toric variety associated to a permutohedron P and it has the symmetry of the symmetric group \mathfrak{S}_n . In joint work with Horiguchi-Shareshin-Song, we have proved that the ring of invariants in $H^*(X; \mathbb{Q})$ under a Young subgroup \mathfrak{S}_λ is isomorphic to the cohomology ring of the toric orbifold associated to the *quotient* of P by \mathfrak{S}_λ . I feel that this result is the tip of iceberg of much more general result and try to extend our result.

(2) Research on mathematics related to Hessenberg varieties which are subvarieties of flag varieties. Several years ago, Hiraku Abe, Megumi Harada, Tatsuya Horiguchi and I studied the cohomology rings of Hessenberg varieties. Later, it turned out that this work is related to hyperplane arrangements and Stanley- Stembridge conjecture on graph theory. My goal is to solve the Stanley-Stembridge conjecture and I am working with Anton Ayzenberg and Takashi Sato.

(3) I am working on torus orbit closures in flag varieties with Eunjeong Lee and Seonjeong Park for these four years and will continue this project.

(4) I am working on a project with Anton Ayzenberg on the topology of torus actions of complexity one. Torus actions of complexity zero are well studied, where toric manifolds provide examples of torus actions of complexity zero. Complexity one case is the next natural target to attack and recent work shows that one can expect beautiful results.