Research Achievements

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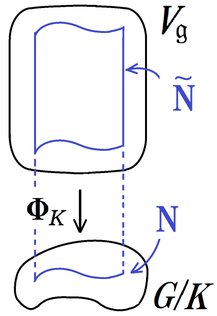
One way to study submanifolds in a compact symmetric space G/K is to consider their lifts into a certain infinite dimensional Hilbert space. Let $V_{\mathfrak{g}} = L^2([0,1], \mathfrak{g})$ denote the Hilbert space all L^2-paths from $[0,1]$ to the Lie algebra $\mathfrak{g}$ of G. C.-L. Terng and G. Thorbergsson introduced a natural Riemannian submersion $\Phi_K : V_{\mathfrak{g}} \to G/K$, called the *parallel transport map*. For a closed submanifold N of G/K its inverse image $\tilde{N} := \Phi_K^{-1}(N)$ is a *proper Fredholm* submanifold of $V_{\mathfrak{g}}$ and its shape operators are compact self-adjoint operators. Although $\tilde{N}$ is infinite dimensional, many techniques in the finite dimensional Euclidean case are still valid due to linearity of the Hilbert space $V_{\mathfrak{g}}$. Using those techniques they studied submanifold geometry in symmetric spaces. It is a fundamental problem to show the geometrical relation between N and $\tilde{N}$.



In my research, I studied the geometrical relation between N and $\tilde{N}$, especially the relation concerning the *symmetries* of minimal submanifolds.

In [2] I first showed a relational formula between the shape operators of N and $\tilde{N}$. Then I showed a necessary and sufficient condition for $\tilde{N}$ to be a totally geodesic PF submanifold of $V_{\mathfrak{g}}$. Moreover I extended the concept of *weakly reflective* submanifolds (Ikawa-Sakai-Tasaki 2009) to a class of PF submanifolds in Hilbert spaces. Here a weakly reflective submanifold is a minimal submanifold such that for each normal vector ξ there exists an isometry which preserves N and reverses ξ. Then I showed that each fiber of Φ_K is a weakly reflective PF submanifold of $V_{\mathfrak{g}}$. Moreover I showed that if N is a weakly reflective submanifold of G/K then $\tilde{N}$ is a weakly reflective PF submanifold of $V_{\mathfrak{g}}$. Using these results I showed many examples of infinite dimensional weakly reflective PF submanifold of $V_{\mathfrak{g}}$ which are not totally geodesic.

In [3], using the formula for the shape operator obtained in [2] I first showed a relational formula for the principal curvatures of N and $\tilde{N}$ under the assumption that N is a curvature-adapted submanifold. This formula corrects the formula given by N. Koike and the proof is different from his. Next, using this relational formula I studied the relation between the austere properties of N and $\tilde{N}$. Here a submanifold is called *austere* if the set of principal curvatures in the direction of each normal vector is invariant under the multiplication by (-1). By definition we have the relation "weakly reflective $\Rightarrow$ austere $\Rightarrow$ minimal". Although the principal curvatures of $\tilde{N}$ are complicated in general, I supposed that G/K is the standard sphere and showed that N is austere if and only of $\tilde{N}$ is austere. Moreover I studied the relation between the *arid* properties of N and $\tilde{N}$. Here arid submanifolds were introduced by Y. Taketomi and they generalize weakly reflective submanifolds. I showed that if N is arid then $\tilde{N}$ is also arid. Using those results I gave examples of austere PF submanifolds, and arid PF submanifolds which are not austere.

In [4] I extended the results of [2] to the case that G/K is a compact isotropy irreducible Riemannian homogeneous space. This result is based on discussions with Professors E. Heintze, J. H. Eschenburg and T. Sakai during my visit to the University of Augsburg.

In [5], as an extension of [3], I studied the relation between the austere properties of N and $\tilde{N}$ under the assumption that N is an orbit of a *Hermann action*. Here a Hermann action is defined as an isometric action of a symmetric subgroup of the isometry group. It is hyperpolar and all of its orbits are curvature-adapted submanifolds. In this case $\tilde{N}$ is expressed as an orbit of a path group action. First I introduced a hierarchy of curvature-adapted submanifolds in G/K and refined the formula for the principal curvatures given in [3]. Using this formula I derived an explicit formula for the principal curvatures of $\tilde{N}$ under the assumption that N is an orbit of a Hermann action. This formula unifies and generalizes some results by C.-L. Terng, U. Pinkall, G. Thorbergsson and N. Koike. Using this formula I showed that if N is austere then $\tilde{N}$ is also austere and that the converse is not true by showing a counterexample.

In [6] (preprint) I discovered and formulated a canonical isomorphism of path space and extended the results of [5] to the case of σ*-actions*. Furthermore I showed that all known computational results of principal curvatures of PF submanifolds by Terng, Pinkall-Thorbergsson, King-Terng are understood as special cases of the general formula given in [5].