On left-invariant metrics

For a Lie group G, denote by $\mathcal{M}_L(G)$ the set of all left-invariant metrics on G. I have shown that if an orbit of the action $\mathbb{R}_{>0} \times \mathrm{Aut}(G) \curvearrowright \mathcal{M}_L(G)$ is an isolated orbit, then any left-invariant metrics contained in the isolated orbit are Ricci soliton.

The Ricci soliton is defined as a metric which gives a self-similar solution of the Ricci flow equation. Recent studies have revealed that if the orbit $\mathbb{R}_{>0} \times \mathrm{Aut}(G).\langle , \rangle$ is an isolated orbit, then $\langle , \rangle$ gives a self-similar solution not only the Ricci flow equation but also various metric evolution equations. Hence I study these left-invariant metrics more deeply as follows:

- More examples of Lie groups G whose $\mathbb{R}_{>0} \times \mathrm{Aut}(G)$-actions have isolated orbits are required since they give nice examples for studying self-similar solutions of metric evolution equations. To find examples, 2-step nilpotent Lie groups are nice targets since their automorphism groups are relatively simple.
- A goal of my research is to classify Lie groups G which admit isolated $\mathbb{R}_{>0} \times \mathrm{Aut}(G)$-orbits. The generalized Alekseevskii conjecture has been proved by Böhm and Lafuente. As a consequence, a Lie group G which admits a left-invariant Ricci soliton $\langle , \rangle$ is isometric to a solvmanifold or a product space of a compact Einstein Riemannian Lie group and a flat abelian group. Recently, I have found some algebraic obstruction for an $\mathbb{R}_{>0} \times \mathrm{Aut}(G)$-orbit to become an isolated orbit. I try to classify solveble Lie groups G which admit isolated $\mathbb{R}_{>0} \times \mathrm{Aut}(G)$-orbits by applying the obstruction.

On arid submanifolds

I have generalized the essence of isolated orbit to arbitrary submanifolds as arid submanifolds. My future works on arid submanifolds will divide into two cases (i)homogeneous cases, (ii) inhomogeneous cases:

(i) One can see that a homogeneous arid submanifold is an isolated orbit of some isometric action. One of the easy examples of isolated orbits is a non-principal orbit of a cohomogeneity one action. These isolated orbits are characterized as

  a homogeneous submanifold whose full slice representation on the unit normal sphere is transitive. $(\star)$

  A goal is to classify submanifolds which satisfy $(\star)$ in Riemannian symmetric spaces. Cohomogeneity one actions on the space forms are well-understood. Hence, firstly, I observe $(\star)$-submanifolds in the space forms, and try to classify in the general settings.

(ii) No example of complete inhomogeneous arid submanifolds in homogeneous spaces seems to be known. One of the nice examples of inhomogeneous submanifolds are some isoparametric hypersurfaces in the sphere. Firstly, I want to investigate whether the focal submanifolds of isoparametric hypersurfaces in the sphere are arid submanifolds or not.