The plan of researches

The expansion law of the Friedmann universe model is determined by three parameters; Hubble parameter, deceleration parameter and cosmological constant. In order to determine the deceleration parameter q, we have to observe the apparent luminosity (magnitude) m and the redshift z of a lot of celestial bodies. We can obtain the m-z relation of our universe from these observations. Since we can analytically obtain the m-z relation of the Friedmann universe model including q as a parameter, we can accomplish the purpose by finding the value of q which realizes the observed relation.

In the real universe, galaxies and clusters of galaxies do not distribute homogeneously. When a light ray passes near a galaxy, it suffers gravitational lens effect. This effect results in bending of the light ray and an amplification of the brightness. Therefore, we cannot make such a simple comparison as mentioned above.

There are several researches about the effects of inhomogeneities on the $m \cdot z$ relation. Only weak lensing effects are considered. This consideration is valid only if the light ray does not pass very close to the lens object. But, it is a well known fact that a light ray which come from distant celestial body $(z \gtrsim 1)$ does pass very close to the lens objects at least once in its own history. We should not apply the previous results since we have to observe distant celestial bodies $(z \gtrsim 1)$ in order to determine q using the $m \cdot z$ relation.

In order to obtain a m-z relation, we have to study a cross sectional area of light beam which comes from a celestial body. To be more precise, we solve the geodesic equations and the geodesic deviation equations. It is possible to solve analytically these equations in the case of the Friedmann universe model but we have to rely on the numerical analysis in the case of an inhomogeous universe.

In the future studies, we construct a simple universe model where black holes are embedded in the Friedmann universe model and obtain the m-z relation of this model. We can clarify the effects of the local inhomogeneities.

As a special case of the above model, we can construct the universe model filled with extremely small black holes. It is a very interesting problem whether the conventional m - z relation of the Friedmann universe model is realized in this model. In case that an embedded black hole region is much larger than the thickness of light beam, we can study changes in the sectional area by solving the geodesic deviation equations. On the other hand, in this black hole dominated universe, there exist a lot of black holes in a beam. In order to study the area of the beam, we have to find each trajectory of light rays of which the beam consists. By using this method, non-trivial configuration of light rays will appear when one of the light rays pass very near a black hole.