Summary of the previous studies

In the framework of the general relativity, it became possible to study the universe itself as a subject of physics. In our universe, galaxies and clusters of galaxies does not distribute homogeneously. But after averaging over large scales, we can regard the matter distribution to be homogeneous. The Robertson-Walker solution is a spatially homogeneous and isotropic one of the Einstein field equation. The Friedmann universe model is favorable since it has succeeded in explaining various observational facts. This model brings a new feature that universe is expanding. Taking a region arbitrarily in an inhomogeneous expanding universe, the expansion rate depends on whether the universe is exactly homogeneous or approximately homogeneous.

There are several researches about the volume expansion of an inhomogeneous universe. Inhomogeneities are treated as perturbations from the Friedmann universe model. Since the perturbations have growing modes, it is general that inhomogeneities become non-perturbative. In paper 1, we construct an inhomogeneous universe model, called Swiss-cheese (SC) universe model, and study the effects of inhomogeneities on the volume expansion.

Our Swiss-cheese universe model is constructed by embedding the Tolman-Bondi (TB) solutions in the Freedmann universe model. The TB solution describes spherically symmetric spacetime with dust fluid, therefore SC universe model has locally inhomogeneous matter distribution. This model has two features:

- This model, as a whole, is an exact solution of the Einstein field equation.
- The inhomogeneities can be treated perturbatively in the early stage of its evolution.

The second feature enables us to compare the results with the previous perturbation-based studies.

Investigating the volume expansion rate in this model, we obtained the following two results:

- As long as the inhomogeneities can be treated as perturbations, expansion rate is decelerate.
- The effect of inhomogeneities vanishes as the evolution goes on.

The first result agrees with the previous perturbation-based studies. We found that the volume itself experiences the contracting phase as a result of early deceleration and turns to expand as the universe evolves.