My Results

I am the first grade student of the doctor course at the Department of Mathematics, Osaka City University.

The result I have published is a master thesis of my own written in 1991. So I explain here the thesis “Hausdorff dimension and Self-similar set”.

The thesis has an expository part dealing with Hausdorff dimension and self-similar sets. They are very useful techqnics in Fractal Geometry. It also has explanations of some preliminaries and one of the newest result about self-similar set at that time.

The self-similar sets are built up of pieces geometrically similar to the entire set but on a smaller scale. More accuratelly, let K be a compact set in a complete metric space X. We call the set K is self-similar, or K has self-similarity if for a finite set of contractions $\{S_1, \ldots, S_n\}$ of X, the union $\cup_{i=1}^{n} S_i(K)$ equal to K. Some typical fractals such as Koch curves, Sierpinski gaskets, Cantor sets and so on, are self-similar.

The Hausdorff dimension is a quantity representing complexity of a fractal set. The Hausdorff dimension of a subset in the n dimensional Euclidean space is n provided that it has positive Lebesgue measure. In general, Lebesgue measure of fractal set is 0 or ∞. Accordingly, Lebesgue measure is no use to investigate fractal sets. On ther other hand, for each fractal set K, there exists a non-negative real number s such that s dimensional Hausdorff measure has a finite measure on K. This s is the Hausdorff dimension of K. It is very important to measure fractal sets.

In general, the Hausdorff dimension is hard to calculate. However Hutchinson shows in his paper of 1981 if the contractions twhich defines a self-similar set are compositions of dilation, rotation and translation, and they satisfy “Open set condition”, then the Hausdorff dimention of the set coincides the similarity dimension. The similarity dimension has advantage of being easily calculable. In Hutchinson’s paper, a self-similar set K is defined as the image of continuous mapping of an Alphbet space. An Alphbet space means a countable product space of $\{1, \ldots, n\}$. Then the main results are derived by using the geometric measure theory in the Alphbet space.

Handa deals with the case that the contractions are C^1 class in his preprint in 1990. He also makes use of geometric measure theory in the Alphbet space, in addition ergodic theory and statistical mechanics. Then he shows that the Hausdorff dimension can be expressed by a kind of entropy.

My master thesis finishes at giving the Handa’s result. When I was the second grade student of Master course at Kyushu University, I tried to improve the results of Hutchinson but I could not. For that reason, the thesis has not original work of mine.