Plans for my research

1. Theme

I will mainly work on a research of estimating a number of holomorphic sections over a holomorphic family of Riemann surfaces.

2. Background and Significance of the problem

This estimation is closely related to the Mordell conjecture in Number Theory, and also has connection with the Farmet conjecture. The Mordell conjecture asserts that an algebraic equation over rational number field, $P(x, y) = 0$, has only finitely many solutions of rational numbers if the genus of the Riemann surface defined by this equation is more than 1, which is proved affirmatively by Faltings.

If you change, in the statement of the conjecture, "algebraic equation over rational number field" to "functioal equation over a functional field" and "solutions of rational numbers" to "solutions of rational functions" , then you obtain a new problem what is called the Mordell conjecture over a function field, which is proved by Grauert and Manin before Faltings achieved his proof of the Mordell conjecture in Number theory. Grauert's and Manin's ideas in their proof gave Faltings some light and mile-stones to solve his arithmetic problems, and also they are thought of as a prototype model of arithmetic geometry.

Faltings' proof lets us know the finitenass of a number of solutions, but nothing about the estimation how many solutions there are. The problem of the estimation has not been solved. This is a subject I am going to pursue. If some results about it come up, we would expect some applications to a similar problem in the case of arithmetic version.

3. Methods to investigate and its advantage

The Mordell conjecture over a functional field can be restated by words of complex manifold theory. It is equivalent to say that "the number of holomorphic sections for a holomorphic family of Riemann surfaces with genus more than one is finite". Therefore, the estimating the number of solutions is the same as the estimating the number of non-trivial holomorphic sections. Namely we can deal with the conjecture through complex geometric approach.

Because we can investigate a holomorphic family of Riemann surfaces in detailed and calculable way by using theory of Moduli spaces and Teichmüller spaces, it is possible to analyze the problem quantitatively. Moreover, Imayoshi-Shiga rigidity theorem says "holomorphic families and its holomorphic sections are determined by only topological information called monodromy ". Therefore we could have the estimation if we get information of its monodromy. Techniques of 2 and 3 dimensional hyperbolic manifolds and Kleinian groups are also available to do it.

4. Concrete targets for my research

I am going to investigate the following subjects through explicit typical examples.

(1) Classification of monodromy of a holomorphic family of Rieman surfaces

(2) Information to determine the class of monodromy (e.g. length of geodesics)

(3) Estimating the number of monodromies

5. Related topics

A monodromy above has correspondence with an element of the mapping class group of a surface, and it is closely related to knot theory and braid theory. For the last several month, I have workd on above problems with partly using ideas of C.T.McMullen's paper "From dynamics on surfaces to rational points on curves" as a guide. Through this study, I have had a thought that we shuoud obtain more knowlede concerning the Geometry of Moduli spaces. So, I will continue to pursue my target through above approach and newly start to investigate the geometric structure of Moduli spaces with complex analytic method.