Research plan

1. Main subject is the study of tilting modules. Let A be a finite-dimensional algebra over an algebraically closed field and T a tilting module over A. Then the number of isomorphism classes of simple A-modules is equal to that of indecomposable direct summands of T. Now, there has been a problem whether this property may replace one of the conditions of a tilting module that there is an exact sequence: $0 \to A \to T_0 \to \cdots \to T_r \to 0$, where $T_0, \ldots, T_r \in \operatorname{add} T$. I will first investigate under which conditions this is true. If T is a tilting module in the classical sense, this has been known to be true without conditions.
2. Rickard gave an affirmative answer to Broué's abelian defect group conjecture in the cyclic defect case, which actually gave a derived equivalence classification of a certain class of representation-finite self-injective algebras. And Asashiba, as a generalization of Rickard's result, classified the representation-finite self-injective algebras up to derived equivalences by making use of the covering technique. Moreover he worked up the derived equivalence classification of a class of self-injective algebras including representation-infinite self-injective algebras. The class consists of twisted multifold extensions of piecewise hereditary algebras of tree type, namely, those algebras of the form $\hat{A}/\langle\hat{\phi}\nu^n\rangle$, where $\hat{A}$ is the repetitive algebra of a piecewise hereditary algebra A of tree type, ϕ is an automorphism of A, ν is the Nakayama automorphism of $\hat{A}$, and n is a natural number. Now the second purpose is to study the derived equivalence classification of a wider class of self-injective algebras obtained by a rigid automorphism of $\hat{A}$ (not necessarily induced by that of A). The idea is to set up an invariant H_A for a self-injective algebra A in this class which is compatible with the stable equivalence of Morita type, to derive that given self-injective algebras A and B, the following three conditions are equivalent to each other: A and B are stably Morita type equivalent; the invariants H_A and H_B are the same; A and B are derived equivalent. So, if this is true, the derived equivalence can be characterized by the invariants H_A in the class of self-injective algebras A.
3. Also I intend to study Lie theory via the derived categories, being based on the book edited by Ringel and some others.